Investigation of Forced Convective Heat Transfer in Nano fluids

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Abstract: The dissipation produced by viscous term when a two-dimensional flat plate reached different temperatures is taken into account in the theoretical analysis of heat transfer in the laminar two-dimensional flows of different nanofluids mentioned in this research. The stable incompressible flow equations were transformed into a nonlinear ordinary differential equation using a similarity variable (ODE). These equations were numerically solved by replacing the partial derivatives in the implicit finite difference method with the appropriate central difference patterns, and the algebraic equations that resulted were then linearized using Newton's method. Finally, the block-tridiagonal-elimination approach was used to solve the linear system. The three types of nanoparticles that were considered were Cu-water, Al2O3-water, and TiO2-water. In the analysis, the mathematical application Mathematica was used. At various wall temperatures, various nanoparticle kinds, values for the nanoparticle volume fraction, and the Eckart and Prandtl numbers were examined. These parameters' effects on flow behaviour, local skin friction, Nusselt number, velocity profiles, and temperature profiles were examined and presented. It is determined that these variables have an impact on heat transfer parameters and fluid flow behaviour, particularly nanoparticle concentration. The rate of heat transmission was improved by the presence of nanoparticles, and the type of particle has a big impact on the improvement of heat transfer.

Keywords: Nanofluid, Flat Plate, Heat Transfer, Viscous Dissipation, Wall Temperature

1. Introduction

Recent research has shown that nanofluids can boost thermal conductivities and can enhance fluids' capacity to transmit heat, increasing energy efficiency. Nanofluids scatter some nanoscale components in conventional flow. Although these nanoparticles' primary flow is a liquid or gas, their sizes and shapes change depending on their application. The flow is analysed using the two phase mixture theory. The most often utilised materials as nanoparticles are oxide ceramics, metal oxides (alumina, silica, and titania), and metal carbides (SiC) (Al2O3, CuO). First to use the term "nanofluid" was Choi [1].

He concluded that the presence of nanoparticles gives a significant enhancement of their properties. Many of the published researches on nanofluids were concerned with

their behavior. To enhance the thermal properties of such fluidflow, nanoscale particles are being dispersed in a base fluid [2-4]. The results showed that thermal conductivity increased by adding very small amounts of concentration (less than 1% by volume). Nield and Kuznetsov [5] studied using Buongiorno model the free boundary-layer flow of a nanofluid past a vertical plate. Xuan and Roetzel [6] were the first researchers to indicate a mechanism for heat transfer in nanofluids. Dual solutions have obtained by [7] when free stream and the plate move in the opposite directions.

Flow of nanofluid past a fixed or moving flat plate was studied numerically by Bachok et al. [8]. Three different types of metallic or nonmetallic nanoparticles were solved numerically by [9-11]. They deduced that the existence of nanoparticles into the base flow causes an increase in the skin friction and heat transfer coefficients. This paper

investigates numerically 2-D steady flow of nanofluids past a horizontal flat plate embedded in the water-based nanofluid taking into account viscous term and convection of heat transfer. Eckert number is used to characterize viscous thermal dissipation of convection. If the viscous thermal dissipation is ignored then the Eckert number is regarded as zero. Flow and transfer processes can be modeled mathematically by complex systems of equations, which are often non-linear due to both the complexity of the problem and the number of physical variable. There are several ways to solve these differential equations, such as analytical and numerical methods. Mass, momentum and energy conservation equations are transformed using the similarity transformations to a nonlinear Ordinary Differential Equation (ODE), and then the resulting equations are solved using numerical method to give a complete picture of the proposed problem. Three types of nanoparticles in the water based fluid are considered. The flow parameters and heat transfer are studied for various types, values of the nanoparticle, Pr, Ec, and wall temperature. The effects of these parameters on the flow behaviour and mainly on the local skin friction and heat transfer coefficient are investigated.

2. Mathematical Formulation

To provide a reasonable solution of the laminar 2-D equations the following assumptions are considered;

- 1. The flow is assumed as an ideal water-mixture of water and nanoparticles with zero pressure gradients.
- 2. The nanoparticles are spherical shape, rigid and uniformly distributed.
- 3. Equal velocities between the base fluid and the nanoparticles.
- 4. The base fluid and the nanoparticles have the same temperature.

Using the above assumptions the basic equations can be written as follows:

where, x and y are the coordinates, while, *u* and *v* are the velocity components in these directions respectively. While, *p*, *T*, μ_{nf} , ρ_{nf} , and α_{nf} are the pressure, temperature, dynamic viscosity, density and the thermal diffusivity of the nanofluid respectively, which are defined by,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = v_{nf} \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} \right)$$
(2)

$$\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{v_{nf}}{C^{pnf}} \left(\frac{\partial u}{\partial y} \right)^2$$
(3)

 $\rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s, \ \mu_{nf} = \frac{\mu_f}{(1 - \varphi)^{2.5}}, \ \alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}}$

$$(\rho C_{\rho})_{nf} = (1 - \varphi) (\rho C_{\rho})_{f} + \varphi (\rho C_{\rho})_{s}$$
(4)

$$\frac{K_{nf}}{K_{f}} = \frac{(K_{s} + 2K_{f}) - 2\varphi(K_{f} - K_{s})}{(K_{s} + 2K_{f}) + \varphi(K_{f} - K_{s})}$$
(5)

Where, ϕ is the nanoparticle concentration.

- Inlet and free boundary conditions for the fluid flow are;



Figure 1. Flow and convective boundary heat transfer.

$$u = v = 0, \ T = T_w(x), \ at \quad y = 0,$$
$$u \to U \ as \ T \to T_{\infty}$$
(6)

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Assuming that the relation between ambient nanofluid and the wall surface temperatures T_{∞} , T_{w} respectively is,

$$T_w(x) = A x^n + T_\infty \tag{7}$$

Introducing the following similarity variables in Eqs. (1-3) with the boundary conditions (6)

$$\eta = y_{\sqrt{U/xv_f}}, \quad (x, y) = f(\eta) \quad \sqrt{Uxv_f},$$
$$\theta(\eta) = (T - T_{\infty}) / (T_w(x) - T_{\infty}), \quad (8)$$

Where, V_f is the kinematic viscosity of the fluid fraction. The stream function is common given by,

$$u = \partial \psi / \partial y, v = -\partial \psi / \partial x \quad , \tag{9}$$

Substituting Eqs. (8-9) into Eqs. (2-3) and using the transform [12] to obtain the following uncoupled equations,

$$f'' + 0.5(1-\phi)^{2.5}(1-\phi+\phi\rho_s / \rho_f) f f' = 0,$$
(10)

$$\theta' + P_r / (K_{nf} / K_f)(1 - \phi + \phi (\rho C_p)_s / (\rho C_p)_f)$$

$$\times (0.5f\theta' + Ec(f')^2 - nf'\theta) = 0, \tag{11}$$

Eqns. (10-11) are subjected to,

$$f(0) = f'(0) = 0, \ \theta(0) = 1, \ f'(\infty) = 1, \ \theta(\infty) = 0$$
(12)

The local skin friction coefficient is defined by,

$$C_f = \tau_w / (U^2 \rho_f) \tag{13}$$

While the plate surface shear stress is defined by,

$$\tau_w = \mu_{nf} \left(\partial u \,/\, \partial y \right)_{y=0}$$

Pr, Nu are the Prandtl number and the local Nusselt number which are defined as,

$$\Pr = v_f / \alpha_f, Nu = xq_w / k_f (T_w - T_\infty), \qquad (14)$$

Where, q_w is the heat flux from the plate,

$$q_w = -K_{nf} \left(\partial T / \partial y\right)_{y=0},\tag{15}$$

Substituting Eqn. (8 and 15) into Eqs. (13, 14) gives,

$$R_{ex}^{-1/2}C_f = f'(0) / (1-\phi)^{2.5}, \quad R_{ex}^{-1/2}Nu = -\theta'(0) K_{nf} / K_f \quad (16)$$

Where, $Re_x = Ux/V_{nf}$ is the local Reynolds number.

The Eckert number is defined as the ratio of a flow's kinetic energy to the boundary layer enthalpy difference.

$$E_{c} = U^{2} / C_{p} (T_{w} - T_{m}), \qquad (17)$$

The Eckert number is regarded as zero when the viscous thermal dissipation is neglected.

3. Mathematical Solutions

A set of coupled equations is the transformation of the governing nonlinear PDE using a similarity variable. This set of equations is solved numerically using the method reported in [13-15] in which the partial derivatives are replaced by appropriate central differences patterns and using Newton's method to linearize the resulting algebraic equations. Finally, the block-tridiagonal-elimination technique is used to solve that linear system.

4. Mathematical Validation

Tables 1 to 3, show the values of temperature gradient at wall the $-\theta'$ (0). These values are compared with that obtained by [15]. To validate the numerical results the same boundary conditions are used. Therefore a flow without any nanoparticles is tested. In Table 1, the values of $-\theta'(0)$ are calculated at Pr = 0.7 for various values of Ec for variable values of wall temperature index (n = 1, 2, 3 and 4).

In Table 2, the values of $-\theta'(0)$ are calculated for Ec = 0.5 for various values of Pr. In Table 3, the values of $-\theta$ (0) are calculated for n = 3 for various values of Pr with various values of Ec. From Table 1, it is seen that the values of $-\theta'(0)$ decrease when Ec increases for all tested values of n, the results obtained show a good agreement with the published data.

Table 1. Values of $-\theta'(0)$ for Pr = 0.7 and variable temperature index, n.

Method	Present	Data [15]	Present	Data [15]	Present	Data [15]
n	Ec = 0.1		Ec = 0.5		Ec = 0.7	
1	0.471658	0.471081	0.436675	0.433778	0.419183	0.415127
2	0.576926	0.576896	0.546819	0.544484	0.531765	0.528278
3	0.654004	0.654579	0.626875	0.625153	0.611444	0.610440
4	0.716296	0.717506	0.691264	0.690181	0.67489	0.676519

Table 2. Values of $-\theta'(0)$ for Ec = 0.5 for temperature index, n.

Method	Present	Data [15]	Present	Data [15]	Present	Data [15]
n	Pr = 0.7		Pr = 3		Pr = 5	
1	0.436675	0.433778	0.660545	0.645443	0.746864	0.729576
2	0.546819	0.544484	0.841009	0.837255	0.977085	0.964844
3	0.626875	0.625153	0.979378	0.974597	1.13878	1.13158
4	0.691264	0.690181	1.08606	1.084292	0.67489	0.676519

Table 3. Values of $-\theta'(0)$ for n = 3 for variable Ec nd Pr.

Method	Present	Data [15]	Present	Data [15]	Present	Data [15]	
Ec	Pr = 0.7		Pr = 3		Pr = 5		
0.1	0.654004	0.654579	1.05296	1.057992	1.24108	1.250237	
0.3	0.64044	0.639866	1.01617	1.016295	1.18993	1.190909	
0.5	0.626875	0.625153	0.979378	0.974597	1.13878	1.13158	
0.7	0.613311	0.61044	0.942587	0.9329	1.08763	1.072252	

5. Results and Discussion

Solution of the system of equations (10 and 11) with the help of (12) is obtained. The effect of the nanoparticles volume fraction ϕ , Prandtl number Pr, Eckart number Ec and wall temperature index n on the flow characteristics are discussed and analyzed for three different types of nanofluids

Cu-water, Al2O3-water, and TiO2 -water as working fluid. The effect of solid concentration φ is investigated in the range of $0 \le \varphi \le 0.2$, Prandtl number number range $0.004 \le$ Pr ≤ 6.2 , Eckart number range $0 \le \text{Ec} \le 1$ and n = 0, 1, 2, 3, 4 and 5 is investigated in details for Cu-water nanofluids.

Figure 2 shows the temperature variation past flat plate for different values of Pr. From the figure it is observed that increasing Pr, the temperature profile decreases for different

values of nanoparticles concentration for fixed Pr, Ec. Also the figure shows that increasing the values of φ the temperature increases as a result of heat gained from the nanoparticles. In case of zero nanoparticle concentration Figure 2 the predicted results are exactly the same as data reported in [15].



Figure 2. Effect of nanoparticles concentration on temperature distribution.

Figure 3 shows that the temperature decreases as η increases for specific wall temperature (n = constant) and also decreases as n increases. Also the figure shows the effect of the presence of nanoparticles, it is observed that by increasing the values of φ , the temperature profile increases for variable flat plate temperature index (n = 0, 1, 2, 3, 4, 5) for fixed Pr and Ec.



Figure 3. Temperature variations for different values of n.

The results obtained for temperature variation for various values of the Eckert number at constant values of Pr, n and ϕ are shown in Figure 4. It is clear from the figure that as Ec increases the temperature distribution increases. In the case of zero Ec, this means that viscous thermal dissipation is ignored.



Figure 4. Temperature distributions for different values of Ec.

Figures 5-6 present the variation of skin friction coefficient $(\text{Rex}^{1/2}, \text{ C}_f)$ and the Nusselt number $(\text{Rex}^{-1/2}, \text{ Nu})$ in case of the presence of nanoparticle for the three tested working

fluids. It is noticed from the figures that both numbers of $\breve{C_{\rm f}}$

and Nu increase when increasing the values of φ . Hence, more particles are suspended and thermal conductivity of nanoparticles increases. On the other side, figures indicate that more fluid is heated for higher values of φ . Also the

figures show that the lowest skin friction coefficient is obtained for Al_2O_3 , on the other side the lowest value of the Nusselt number is obtained for TiO₂ this is because TiO₂ has the lowest thermal conductivity compared with Cu and Al_2O_3 .

While Figures 7-9 show the variation of Nusselt number in case of using Cu-water as working fluid for different values of Pr, n and Ec respectively. It is noticed that Nusselt number increases as Pr and n increases and decreases as Ec increases.

The transverse component of the flow velocity is shown in Figure 10. The results show that small values of φ has a small effects on transverse component of the velocity.



Figure 5. Variation of, C_f , with φ for different types of nanoparticles.



Figure 6. Variation of the Nu, with φ for different types of nanoparticles.



Figure 7. Effect of Pr on Nu number at constant values of Ec and n.



Figure 8. Effect of n on Nu number at constant values of Ec and Pr.



Figure 9. Effect of Ec on Nu number at constant values of Pr and n.



Figure 10. Effect of ϕ on velocity variation past flat plate at constant values of Pr, Ec and n.

6. Conclusions

In the current work, expected results are achieved for a variety of unique parameters and are described. The following conclusions are possible:

Due to the presence of nanoparticles in the base fluid, the numbers of Cf and Nu increase as the volume percent of nanoparticles rises. Additionally, as the quantity of nanoparticles grew, so did the rate of heat transfer. The efficiency of heat transmission is significantly influenced by the kind of nanofluid utilised. The results showed that the maximum values for both Cf and Nu were obtained while using Cu nanoparticles in water, the base fluid, with a Prandtl number of P r = 6.2. Ec, wall temperature, and Pr number have a significant impact on the thermal boundary layer and heat transport.

Nomenclature

- nf: Nanofluid.
- Re: local Reynolds number.
- Ec: Ecart number.
- f: dimensionless stream-function.
- s: Solid.
- w: Wall.
- φ: nanoparticle volume fraction.
- ψ : Stream function.
- τ: Wall shear stress.

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