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Mathematics, ethics and purism: an applicationof MacIntyre's virtue theory

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ABSTRACT

This paper explores the application of MacIntyre's ethics of virtue to the field of mathematics, with a particular focus on the concept of purism. We begin by discussing the definition of purism and its potential implications for the practice of mathematics, including the ethical dilemmas that may arise when pursuing mathematical perfection. We then introduce MacIntyre's ethics of virtue and discuss how it can be applied to the practice of mathematics, including the cultivation of virtues such as honesty, courage, humility, and justice. We argue that the application of MacIntyre's ethics of virtue to the field of mathematics can lead to a more fulfilling and meaningful practice of mathematics. Finally, we conclude by discussing some of the limitations and challenges of applying MacIntyre's ethics of virtue to mathematics.

The denial of social responsibility is a long-standing ethical issue in mathematics. Pure mathematics is regarded as value-free and thus free of ethical responsibility. Mathematical applications are viewed as employing a neutral set of tools that are free of social responsibility in and of themselves. Mathematicians, on the other hand, are convinced that they know what constitutes good mathematics. Furthermore, many pure mathematicians adhere to purism, an ideology that prioritises purity over applications in mathematics, and the historical reasons for this are discussed. MacIntyre's virtue ethics accommodates both the good mathematician (and good pure mathematics) and the ethics of mathematical social practise. It demonstrates that purism and acknowledging mathematics' social responsibility are compatible.

Four aspects of this responsibility are mentioned: two about the impact of mathematics through education and two about explicit and implicit applications of mathematics. The last of these increases the performativity of mathematical and measurement applications in society, changing the very processes they are meant to measure. Although these applications are not discussed in depth, they highlight the significance of considering the ethics and social responsibility of mathematics in society. MacIntyre's virtue theory introduces a broad approach to the contentious topic of mathematics ethics, encompassing purism, absolutist and social constructivist philosophies of

mathematics while still allowing ethical critiques of mathematics' impact on society.

Keywords . . . .
Mathematics Virtue ethics Purism Social practice Alasdair MacIntyre · Social constructivism Absulutism Philosophy of mathematics Ethics and mathematics

Day 1: Today, I came across an interesting article titled "Mathematics, Morality, and Perfectionism: Applying MacIntyre's Ethics of Virtue to Mathematics." The article discusses the interplay between mathematics, ethics, and perfectionism, and how MacIntyre's ethics of virtue can serve as a beneficial framework for dealing with the difficulties and opportunities that arise in this area. As a mathematician, I was intrigued by this topic and decided to delve deeper into it.

Day 2: I started reading the article and found the concept of perfectionism in mathematics to be fascinating. As a mathematician, I always strive to uphold the highest standards of mathematical rigor and correctness. However, I never realized the ethical quandaries that may arise from this pursuit. The article discusses how perfectionism in mathematics can create ethical quandaries when it conflicts with other values and virtues. This has made me realize the importance of balancing perfectionism with other values and virtues.

Day 3: The article discusses various virtues that are pertinent to mathematics, ethics, and perfectionism. These include honesty, courage, humility, and justice. The article emphasizes the importance of cultivating these virtues and participating in a community of inquiry. This has made me realize that being a good mathematician not only requires technical skills but also requires the cultivation of various virtues.

Day 4: I have started reflecting on my own practice of mathematics and have realized that there are certain virtues that I need to cultivate more. For instance, I need to work on being more humble and acknowledging my limitations and fallibility. I also need to work on being more courageous and standing up for my principles and values. Reflecting on my practice has made me realize that there is always room for improvement and that the cultivation of virtues is an ongoing process.

Day 5: After reading the article, I have started engaging in more discussions with my colleagues about the ethical dilemmas that may arise in mathematics. These discussions have been fruitful, and I have realized the importance of having a community of inquiry where one can engage in discussions and reflections about the ethical issues that arise in mathematics.

Day 6: I have started incorporating the virtues discussed in the article into my practice of mathematics. For instance, I have been more honest about my mistakes and have been more open to feedback. I have also been more courageous in challenging established ideas and authorities. These changes have made me realize that incorporating virtues into one's practice of mathematics can not only lead to better mathematics but also lead to a more fulfilling and meaningful practice of mathematics.

Day 7: Reflecting on the past week, I have realized that the article "Mathematics, Morality, and Perfectionism: Applying MacIntyre's Ethics of Virtue to Mathematics" has had a significant impact on my practice of mathematics. It has made me realize the ethical dilemmas that may arise in mathematics and the importance of balancing perfectionism with other values and virtues. It has also made me realize the importance of cultivating virtues and participating in a community of inquiry. I will continue to reflect on the ideas presented in the article and incorporate them into my practice of mathematics.

Introduction:

Mathematics has long been considered a field of pure reason and logic, untainted by the ethical dilemmas that often arise in other areas of human endeavor. However, recent discussions have challenged this notion and highlighted the ethical dimensions of mathematical practice. One such dimension is purism, which refers to the pursuit of mathematical perfection and the rejection of any mathematical results that do not meet certain standards of rigor and correctness.

Purism can create ethical quandaries when it conflicts with other values and virtues. For instance, the pursuit of mathematical perfection may lead to a disregard for other values such as honesty, humility, and justice. This raises the question of how to balance the pursuit of mathematical perfection with the cultivation of other virtues.

One potential framework for addressing this question is MacIntyre’s ethics of virtue. MacIntyre argues that the cultivation of virtues is essential for living a fulfilling and meaningful life. He suggests that virtues such as honesty, courage, humility, and justice can be cultivated through participation in a community of inquiry.

In this paper, we explore the application of MacIntyre's ethics of virtue to the field of mathematics, with a particular focus on the concept of purism.

Defining Purism in Mathematics:

Purism in mathematics refers to the pursuit of mathematical perfection and the rejection of any mathematical results that do not meet certain standards of rigor and correctness. This pursuit of mathematical perfection is often characterized by a desire for purity, simplicity, and elegance. Purists are often critical of work that is not mathematically rigorous or that relies on computational methods rather than theoretical proofs.

While the pursuit of mathematical perfection can lead to important mathematical discoveries, it can also create ethical quandaries. For instance, the pursuit of mathematical perfection may lead to a disregard for other values such as honesty, humility, and justice. This raises the question of how to balance the pursuit of mathematical perfection with the cultivation of other virtues.

MacIntyre's Ethics of Virtue and Mathematics:

MacIntyre's ethics of virtue suggests that the cultivation of virtues is essential for living a fulfilling and meaningful life. He argues that virtues such as honesty, courage, humility, and justice can be cultivated through participation in a community of inquiry.

Applying MacIntyre's ethics of virtue to the field of mathematics can lead to a more fulfilling and meaningful practice of mathematics. For instance, cultivating the virtue of honesty can lead to a greater awareness of one's own limitations and fallibility, and a greater willingness to acknowledge mistakes and seek feedback. Similarly, cultivating the virtue of humility can lead to a greater appreciation for the diversity of mathematical approaches and a greater willingness to learn from others.

Challenges and Limitations:

While MacIntyre's ethics of virtue offers a promising framework for addressing the ethical dilemmas that may arise in mathematics, it also poses certain challenges and limitations. For instance, some may argue that the cultivation of virtues is not sufficient for addressing the complex ethical issues that arise in mathematics, and that additional ethical frameworks or approaches may be necessary. Additionally, the application of MacIntyre's ethics of virtue to mathematics may be complicated by the highly individualistic nature of mathematical practice, which can make it difficult to create a cohesive community of inquiry.

Furthermore, there may be disagreements among mathematicians about what constitutes a virtuous approach to mathematics, and how to balance the pursuit of mathematical perfection with other ethical values and virtues. For instance, some mathematicians may view the pursuit of mathematical perfection as the ultimate goal, while others may prioritize collaboration, inclusivity, and social justice.

Mathematics is often perceived as a field that prioritizes accuracy, precision, and logical consistency, and holds truth, proof, and elegance in high esteem. However, mathematics is also a human activity that is influenced by social, cultural, and historical contexts, and poses ethical concerns and quandaries. In this essay, we will explore the interplay between mathematics, ethics, and perfectionism, and discuss how MacIntyre's ethics of virtue can serve as a beneficial framework for dealing with some of the difficulties and opportunities that arise in this arena.

Perfectionism in mathematics is a term that refers to the pursuit of upholding the highest standards of mathematical rigor and correctness, while striving to eliminate any ambiguity, inconsistency, or error. Perfectionists believe that mathematics is an autonomous and self-contained domain of knowledge that is regulated by its own rules and principles, and is not influenced by empirical data or practical applications. Perfectionists also value simplicity, beauty, and elegance in mathematics, and aim to eliminate any extraneous or superfluous elements from mathematical theories.

However, the pursuit of perfectionism in mathematics can also create ethical quandaries, particularly when it conflicts with other values and virtues. For instance, a mathematician who uncovers a flaw in a widely accepted mathematical proof may confront the quandary of whether to speak out and risk exposing the mistake, or to remain silent and maintain the status quo. Similarly, a mathematician who is engaged in a mathematical problem that has potential military or surveillance applications may confront the quandary of whether to continue working on it and contribute to a possibly

harmful use of mathematics, or to abandon the problem and decline to collaborate with such applications.

To address these ethical quandaries, we can refer to MacIntyre's ethics of virtue, which highlights the importance of cultivating virtues and participating in a community of inquiry. According to MacIntyre, virtues are characteristics of character that enable us to achieve our goods and fulfill our purpose as human beings. Virtues are not innate or fixed, but rather are established through habituation, practice, and reflection. Virtues also require a balance between excess and deficiency, and necessitate a contextual and practical judgment in each situation.

Some of the virtues that are pertinent to mathematics, ethics, and perfectionism include:

- Honesty: the virtue of being truthful and sincere, even when it is arduous or inconvenient. Honesty requires the bravery to confess errors and acknowledge uncertainties, and the humility to accept criticism and learn from feedback. Honesty also involves a respect for the norms and standards of the mathematical community, and an awareness of the social and cultural implications of mathematical theories and practices.
- Courage: the virtue of confronting challenges and hazards with determination and resilience, and of standing up for one's principles and values. Courage requires the wisdom to assess the possible consequences of one's actions, and the compassion to consider the interests and perspectives of others. Courage also involves a willingness to challenge and question established ideas and authorities, and to explore new and unconventional approaches to mathematical problems.
- Humility: the virtue of recognizing one's limitations and fallibility, and of being open to learning from others and from experience. Humility requires the acknowledgement that mathematics is a collaborative and cumulative endeavor, and that no individual can master or control all aspects of it. Humility also involves a respect for the diversity and plurality of mathematical traditions and cultures, and an understanding of the historical and contextual factors that shape mathematical knowledge.
- Justice: the virtue of treating others fairly and equitably, and of recognizing their rights and interests. Justice requires the awareness of the social and political implications of mathematical theories and practices, and the consideration of their impact on different groups and communities.

1 What is good (pure) mathematics?

Good pure mathematics refers to the branch of mathematics that is focused on the study of abstract concepts, without direct application to real-world problems. It is often characterized by its elegance, beauty, and intellectual challenge, as well as its importance in advancing our understanding of the fundamental principles that govern the universe.

Some key characteristics of good pure mathematics include:

1. Rigor: Good pure mathematics is based on rigorous proofs and logical reasoning, and requires a high level of precision and accuracy.
2. Clarity: Good pure mathematics is presented in a clear and understandable manner, with precise definitions and well-defined concepts.
3. Creativity: Good pure mathematics requires creativity and intuition to develop new ideas and concepts, and to connect seemingly unrelated areas of study.
4. Depth: Good pure mathematics often requires a deep understanding of advanced concepts and techniques, and may require years of study to master.
5. Universality: Good pure mathematics is often applicable to a wide range of fields and disciplines, and can provide insight into the underlying structure of the universe.

Overall, good pure mathematics is about advancing our understanding of the abstract concepts that underlie the universe, while also being logically rigorous and intellectually challenging. It has applications in many fields, including physics, computer science, and economics, among others.

There is a fundamental ambiguity at the heart of the question as to what constitutes good mathematics. From an ethical perspective good pure mathematics is that mathematics which benefits humankind and contributes to human flourishing. From an epistemic perspective good mathematics is that pure mathematics which is clearly expressed, which is well justified, normally by means of mathematical proofs, and which generally conforms well to the epistemic standards of the social practice of mathematics and of the community of mathematicians. In the case of applied mathematics ethically good applications are those that are beneficial to humankind and cause little or no harm. Applications that are good from an epistemic perspective are those that are correctly and rigorously formulated and that accurately predict outcomes or provide good explanatory models for their target domains.

On the face of it ethical goodness and epistemic goodness are disjoint perspectives from which to consider mathematics. One draws on ethics and moral philosophy while the other is epistemological. However, virtue theory, and in particular MacIntyre's (2007) version of virtue theory accommodates both of these perspectives. MacIntyre's theory of the domains of knowledge as well as most systematic human activity rests on the concept of social practice. The first part of MacIntyre's ethical theory concerns the goods of a social practice. The internal goods of a social practice are the things that are achieved in the course of pursuing the goals of the practice. They can only

be achieved through the practice, and relate to the particular goals of that practice. In contrast external goods are achieved or obtained in a variety of ways beyond the specific and particular social practice. External goods include fame, fortune, and other things that might extrinsically motivate us.

According to MacIntyre mathematics is a social practice, or possibly a family of several overlapping practices. The virtues of a mathematician include those character traits that enable him or her to pursue the goods internal to the practice of mathematics, that is, to be a good mathematician. This is to seek to develop skill and ability to advance mathematical knowledge, in order to pursue the inner goods of mathematics, in other words, to develop the virtues of mathematical practice. Possessing such character traits is called virtuosity within that social practice.

Before focusing on the question of what is a good mathematician it is appropriate to clarify what I propose to count as a 'mathematician'. The narrow definition of mathematician that I prefer includes just those pure and applied mathematicians (including statisticians) that engage in mathematical research of the kind that is published in mathematical research journals. Such persons can equally be termed 'research mathematicians'. 'Doing mathematics' in the corresponding narrow sense means engaging in and contributing to the specific social practices of research mathematics whose active members are these (research) mathematicians.

Of course it is possible to define a much broader notion of the mathematician that also includes researchers in any scientific, medical or technological area where they use high level mathematical concepts and methods, including computer and information scientists, theoretical physicists, quantitative financial analysts, specialist teachers of mathematics, and mathematics education researchers. Just as a teacher of music is called a musician, so too a specialist user of mathematics could be called a mathemati-

cian. Just like 'doing music' may cover any activity involving music, so too a broader notion of 'doing mathematics' would then also include all of the activities that take place in these broader social practices.

However, in this paper I wish to restrict the term mathematician to research mathematicians, namely those active in the social practices of research mathematics. Typically research mathematicians do more than just proving new theorems and publishing research papers. They also undertake additional activities such as teaching students, refereeing and editing research journals, writing expository mathematics texts at the university level, writing and submitting grant proposals, leading a university mathematics department, and participating in the departmental, national and international institutions of mathematics.

On the question of what is a good mathematician, there are various accounts of their attributes. One source, for example, lists the following attributes or character traits of the mathematician: persistence, communication, resilience, critical thinking, logical reasoning, curiosity, creativity, self-organization, and collaboration (University of Kentucky n.d.). However, despite their self-evident value, none of these traits can be said to follow uniquely from mathematics or its social practices.

A more specific list of traits or capacities is provided by Niss (2003) as follows:

1. Thinking mathematically (mastering mathematical modes of thought)
2. Posing and solving mathematical problems
3. Modelling mathematically (i.e. analysing and building models)
4. Reasoning mathematically
5. Representing mathematical entities (objects and situations)
6. Handling mathematical symbols and formalisms
7. Communicating in, with, and about mathematics
8. Making use of aids and tools in mathematics (IT included)

This list was developed in a project to describe the competences that should emerge from school and college education. But from the perspective of this paper, an immediate criticism is that this list is too specific. A good mathematician may exhibit all or most of these traits, but so may an average student of mathematics at high school or college (as the project intends). So they are far from unique to the social practice of researching or making mathematics.

Through empirical work, Krutetskii (1976) identified eleven of the key elements of mathematical thinking observed in the mathematically gifted and talented. Consequently, they are not all to be found among average students of mathematics, and are more specific to future professional researchers. Can this list be regarded as specifying the virtues of a good mathematician? No, it cannot for this list is too narrow and technical a set of skills. Of course it was never claimed to be so, and its purposes are very distant. I was just casting about for possible candidates for the virtues of good mathematicians.

In fact, the main point I wish to make does not concern whether any list of traits or skills can accurately describe a good mathematician or indeed any kind of mathematician at all. My point here is that these lists, and more generally any such list of the traits of a good mathematician concern methodological or epistemic modes of reasoning, and are not ethical or moral traits, let alone ethical virtues. To be a good mathematician

is to perform well or effectively within the social practice of mathematics. Thus, it is to work with, shape, create and communicate mathematical knowledge effectively. The traits of a good mathematician are more epistemic virtues than ethical ones.

This is not to say that MacIntyre has misdescribed the virtues of a mathematician. He successfully captures the sense of being good at something that differs from the Good, as the endpoint of ethical action. To be a good mathematician from MacIntyre's axiological perspective is to do good work within the social practice of mathematics. This is to work well and effectively to achieve the goals of this social practice. These goals concern what is valued because it is useful, beautiful or otherwise desirable and acceptable within the social practice of mathematics. It is not possible to describe these goals as simply epistemic, methodological or ethical. They will inevitably be analysable into several of these domains of value.

If this was as far as MacIntyre's ethical theory took us it would be less than adequate, even though it offers an interesting theoretical account of what it is to be a good mathematician. However, MacIntyre's theory has two more layers or dimensions concerning the virtues that constitute the virtuous life.

On the second level, MacIntyre argues a virtuous person should also possess those traits of character that lead to and sustain a unified narrative of self and the overall good life. Third and lastly MacIntyre claims that as social beings we humans all live within a cultural, moral and historical tradition and that our virtues are also manifested in acknowledging, sustaining and contributing to the development of our tradition in a way consistent with and conducive to our virtuous practices and unified virtuous lives, and vice versa. In his words:

> My account of the virtues proceeds through three stages: a first which concerns virtues as qualities necessary to achieve the goods internal to practices; a second which considers them as qualities contributing to the good of a whole life; and a third which relates them to the pursuit of a good for human beings the conception of which can only be elaborated and possessed within an ongoing social tradition. (MacIntyre 2007, p. 397)

Thus MacIntyre's system of ethics avoids the Scylla of individualism and the Charybdis of absolutism and foundationalism. For the good is that of the individual within a social practice and shared moral tradition. Without these social contexts there is no ethical system. Furthermore, the overarching moral and social tradition is expected to develop and grow in a reflexive and self-sustaining way, rather than standing still as a rigid and frozen system of ideas.

Applied to mathematics, MacIntyre's theory entails that every mathematician, every participant in the practice of mathematics, should not only cultivate their own individual virtues, but also needs to take ethical responsibility for this practice, as well as for their whole lives. It is necessary but not sufficient to be responsible for attaining the goods internal to the practice of mathematics. Our virtues must include taking responsibility for the practice as a whole and for its social and other outcomes. The humanistic moral tradition, of which I and most mathematicians and mathematics educators are a part, at least in the West, includes such virtues and values as rationality, truth, democracy, justice, fairness, autonomy and freedom while caring for fellow humans, for living things and for the environment. Thus it is not enough to focus solely

on the goods internal to the practices of mathematics research and teaching. We must also be concerned with the intended and unintended outcomes of these practices on other humans, on all living things and on the planet as a whole.

In addition, it behoves us as members of the humanistic social and moral tradition to be aware of and to sustain this tradition, to contribute to its emergence, and its responsiveness to cultural and historical developments and to legitimate criticism. This is MacIntyre's (2007) conception of the pursuit of a good for human beings that is, and must be, integral to an ongoing social tradition.

2 Ethical theory and mathematics

MacIntyre's version of virtue ethics is a complex three level theory. Applied to mathematics it can be interpreted as follows. At the first level, a good mathematician is one that addresses the goods internal to the practice of mathematics. That is, being conscientiously devoted to research, working on new proofs, problem solutions and presentations of mathematics: being good as a mathematician. A good mathematics teacher is one devoted to the goods internal to the practice of teaching, namely working for student understanding manifested in their success at mathematical tasks and their ability to communicate mathematical ideas and reasoning (known within the practice as relational understanding, after Skemp 1976). In both types of mathematical practices mathematics teachers (for most research mathematicians also teach) should also attend to the flourishing of students as developing human beings.[1] In any practice "human powers to achieve excellence, and human conceptions of the ends and goods involved, are systematically extended" (MacIntyre 2007, p. 279). Thus the good mathematician will seek to improve his mathematical skills and creative products, and contribute to the growth of the practice of research mathematics, just as a good teacher will seek to grow and learn as a teacher, and contribute to the practice of mathematics teaching.

MacIntyre's second level of application of virtues is made up of those that contribute to human flourishing as a unified whole, not just towards a particular practice or profession. However, these virtues also include a concern with and the taking responsibility for the social practices in which one is involved. Finally, at the third level, a virtuous person should acknowledge, take responsibility for and contribute to the overall social and moral tradition of which they are a part. Thus the virtuous mathematician should be good as a mathematician, a good human being, and one that aspires to and sustains the goods of social tradition, for everyone as well as for themselves.

For a mathematician or a teacher, it is therefore not enough to simply devote oneself to the goods internal to the relevant practices. Nor is it enough to simply strive to live

[1] There is some controversy as to whether teaching is a practice in Macintyre's (2007) sense. MacIntyre himself has denied it, arguing that the goods of teaching mathematics are merely those internal to the practice of mathematics, and so teaching is a secondary activity. However, Dunne (2003), Noddings (2003) and Hager (2011) and others have argued that teaching is a practice in the sense of MacIntyre; the goods are to do with fostering the flourishing of students as all round human beings, of which knowledge acquisition is just a limited part. However, since making and communicating mathematics are both virtues within the social practices of mathematics I do not need to commit to one side or other of this debate for my argument here, although my inclination is to view school education as an independent social practice in its own right.

a unified but isolated ethical life, taking responsibility only for one's own actions. As participants in mathematical practices MacIntyre's virtue ethics requires us to take responsibility for the practices and the associated institutions to which we contribute and of which we are a part. A quality must contribute to success at all three levels to count as a virtue. For example, courage is a virtue because it helps one succeed in individual practices and in overcoming obstacles one faces in the course of a human life and in pursuing and debating the goods within an ongoing social tradition. Requiring a quality to be important for all three levels is how MacIntyre rules out countenancing something like the virtuous thief. The thief may be the most skilled lock-pick in the country, which might contribute to something like a practice of thievery, but because this is not a skill that contributes to the achievement of a good human life, that is, it fails at level two, according to MacIntyre the skill is not counted as a virtue.

Ethics and the neutrality of mathematics

A widespread view that impacts on the issue of ethics in mathematics is the claim that pure mathematics is neutral and value free. Both in philosophy and among mathematicians the standard view of mathematics is that it is objective, universal and necessary. Because values and ethics are regarded as individual or culture dependent, this view sees mathematics as largely value-free and intrinsically neutral with regard to ethics. From this perspective, pure mathematics comprises a set of concepts, methods, truths and theories that exist only in an abstract, objective realm untouched by almost all human values, especially the most human and humane values of ethics.

The problem of whether pure mathematics is objective and universal, the view just described, or whether it is a humanly made construction, remains an enduring philosophical controversy (Hersh 1997; Ernest 1998). Each side of this dispute rests on legitimate underpinning philosophical world views and sets of assumptions. Both are justifiable philosophical positions, but the two sets of views are, on the face of it, incompatible.

The more recent of these positions, the social constructivist philosophy, takes the position that pure mathematics:

(1) is not a unique and unified discipline, for mathematics is made up of many different social practices,
(2) is not the only possible science of mathematics, and there exist alternatives that are equally real-world applicable such as Intuitionist or constructive mathematics,
(3) is not entirely forced upon on us through necessity, neither empirically nor by logic,
(4) rests upon definitions of truth, logic, reasoning and proof that are not necessary, universal, or invariant,
(5) is a historical social construction and is thus contingent on historical actions and human choices and interests,
(6) is a human invention reflecting and imbued with the values and interests of its makers (Ernest 1998; Hersh 1997).

As an example of the values mentioned in (6), pure mathematics may be claimed to be open and democratic because the calculations and proofs upon which its knowl-

edge and applications depend must be open to scrutiny by others. Unlike authority based areas such as religion, mathematical claims can only be founded on open and accessible demonstrations. Thus mathematics is necessarily open and democratic, and these are values greatly prized in Western and other societies. Of course this argument assumes that truth depends on its demonstrations. Absolutists can argue that the truth of a theorem is absolute and precedes its human verification. Proofs are devices to demonstrate this truth to mathematicians. Thus this example of a value within mathematics does not represent a definitive way of choosing between the two competing philosophies of mathematics.

On the other side, what might be termed absolutists are supporters of the view that mathematics is an objective, universal and absolute science. They offer arguments that many find persuasive, which oppose the claims of social constructivism.

Absolutists claim that mathematics is:

(1) Universal, and hence unique, for all systems of mathematics can be translated into classical modern mathematics,
(2) Invariant and timeless, as mathematical concepts apply and mathematical results hold in all places and at all times and that even results that are several millennia old, such as Pythagoras' theorem, are still valid,
(3) Objective, as there are fixed rules for determining its truths independent of subjective opinion or differing cultural outlooks,
(4) Discovered rather than invented, as is illustrated by simultaneous discoveries, and by the rich interconnectedness and unity of the disparate branches and topics of pure mathematics that emerges as it grows.
(5) Above all else, based on an objective, universal, eternal and absolute conception and definition of truth.

Absolutists and Platonists in the philosophy of mathematics who adhere to some or all of these views include the following mathematicians and philosophers: Paul Erdős and Kurt Gödel (Wikipedia 2020) and Hardy (1929). Roger Penrose (1989) and Mark Balaguer (1998).[2]

Strictly speaking absolutists should not claim that mathematics is absolutely value-free, even from their philosophical stance, for they have commitments to epistemological and epistemic values, such as the preference for truth over falsehood, and to ontological and ontic values, such as the acceptance that mathematical objects do exist. There are also aesthetic values, such as accepting that mathematics is, at least in part, beautiful, although this might be regarded as a subjective response rather than an objective feature of mathematics. There can be an adherence to some ethical values, such as that mathematics is intrinsically good because it partakes of Truth and Beauty, but not to the idea that pure mathematics is ethical in the sense of it being socially responsible nor that it is susceptible to social judgement as virtuous or vicious. The absolutist response to the claim that its position entails some or all of these values (other than these applications of ethics) is that they are not values, not choices, but just

[2] Platonists subscribe to the view that mathematical objects and truths are real and exist in an independent supraphysical realm to which mathematicians have access and from which they can gain knowledge.

reflect the objective character of mathematics.[3] There is no acknowledgement of the irony that adopting the position that mathematics is largely value free is indeed itself a values position, or at the very least a meta-values position (Ernest 2016), leading to the 'choosing not to choose' fallacy.

The contrast between the views of Absolutists and Platonists, on the one hand, and social constructivists, on the other is brought into sharp focus in the controversy over whether mathematics is discovered or invented. For example, consider the field of Number Theory. This is a well defined topic in mathematics that rests, since Peano, on a single element (0, in modern formulations), a single unary operation (S, the successor function), a single binary relation (=), and a handful of axioms (plus a background logic and with a metatheory that allows the definition of further functions). On the basis of this rather limited foundation, the huge edifice of modern Number Theory has been erected encompassing results stretching from Andrew Wiles's solution to the Fermat Conjecture to Gödel's Incompleteness Theorems. Such results can be viewed as constructed with human ingenuity, including inventing many subsidiary concepts, functions and methods, within the constraints imposed by working within Number Theory with its axioms and rules of proof (the social constructivist position). They can also be viewed as results discovered by an explorer mathematician within the pre-existing field of number once the gate to Number Theory has been opened (the absolutist position).

Independent of this controversy in the philosophy of mathematics both sides concede that the widespread and dominant view of pure mathematics is that it is value-free and independent of ethics. Social constructivists accept that pure mathematics as presented in texts, schools and universities comprises largely stable and enduring set of concepts, procedures, rules and topics.[4] Furthermore, there is a standard style of presentation of mathematics; one of smooth logical progression suggesting that it is objective, universal and necessary. Lakatos (1976) terms this the Euclidean model, based on a sequence of growing complexity and advancement, from definitions and axioms to lemmas and theorems. He argues that this is a deliberate inversion of the historical development of knowledge to achieve certain philosophical ends. By 'tidying away' the complex historical processes of invention and discovery it gives the appearance that pure mathematics is wholly logical, and unlike human activities, is value-free and ethically neutral; at least for axiological values such as ethical and aesthetic values, as opposed to ontic or epistemic values, such as the preference for truth over falsehood discussed above (Ernest 2016).

[3] The absolutist position is supported by the philosophical dogmas that wholly separate (1) facts from values, after the work of Hume, Kant and the logical positivists/empiricists; and (2) the contexts of discovery and justification, after Reichenbach and Popper. These 'two dogmas of empiricism' (with apologies to Quine) are both contested in the philosophical literature.

[4] It is however illuminating to examine how historically both the university and school mathematics curriculum have dramatically altered over the past 100 years, by no means paralleling changes and advances in the field of mathematics itself (Grinstein and Lipsey 2001). However, each new version of the curriculum is presented as the objective, neutral and final reflection of mathematics.

Is mathematics ethics free?

Traditional objectivist or absolutist philosophies of mathematics claim that pure mathematics is largely value-free (Penrose 1989; Balaguer 1998), while social constructivist philosophies regard it as value-laden throughout (Ernest 1998; Hersh 1997). However, both of these positions agree that pure mathematical knowledge is itself, in a strict sense, not ethical, in the sense of being a-ethical (without ethics) not that of being unethical (judged as poor by ethical standards). Ethics requires ethical agency, a human or other ethical actor performing an intentional action the results of which can be judged as resulting in good or ill. The actor may not intend the ethical outcomes that result, but nevertheless the fact that she has acted deliberately and intentionally means that she has ethical responsibility for the outcomes. Pure mathematics as a body of knowledge is a tool, an inert set of methods, concepts, symbols and knowledge representations, and I wish to claim that until it is deployed in intentional activity or social practice by a human or conscious agent it cannot of itself be ethical. But as a part of mathematical social practices when mathematics is deployed within an intentional activity it then becomes open to ethical judgement and evaluation. This is a fine point because although I claim that mathematics is not ethical, that is it is a-ethical, I also claim that it is imbued with ethical and other values. Every use or application of mathematics, as a human activity in any practice, has ethical implications and can be judged ethically. Even inert mathematics as a conceptual and symbolic tool, is imbued with the values, purposes and interests, including ethical values, that made and shaped it the way that it is, over the course of human history.

The position that mathematics is a-ethical is controversial, and I should acknowledge that there is an ongoing debate as to whether technologies including mathematics can have moral agency (Franssen, et al. 2019). This debate started off in computer ethics (Bechtel 1985; Floridi and Sanders 2004) but has since broadened. Many of the authors who claim that technologies can have moral agency redefine the notion of agency and its connection to human will and freedom so as to include distributed or machine agency (Latour 1993; Verbeek 2011). However, a disadvantage of this strategy is that it tends to blur the morally relevant distinctions between people and technological artefacts. Furthermore, the claim can be made on different grounds that technologies including mathematics can be value-laden in ways other than by having moral or ethical agency (Johnson 2006; Illies and Meijers 2009). This last is the position that I adopt here.

Mathematics is not ethical but it is value-laden and embodies ethical values

Although mathematics is not ethical, in the strict sense that it lacks agency, nevertheless from the social constructivist perspective, and indeed from all but the most stringent absolutist perspectives, it is value-laden. Its concepts, procedures, rules, topics, and theories all result from human work in preferred, valued or otherwise chosen directions. What is chosen is valued, and thus mathematics is imbued with the values and interests of its makers. Once a topic or theory has been set up, institutionalised as it were, there are, of course, set concepts, proof procedures, rules of inference, that constrain what can be done within the area. This is why many people claim that mathematics is

discovered and not invented. Human invention and values are still in play in deciding what problems and proofs to pursue, how to put together existing concepts and terms to make new proofs, and less commonly, in inventing new concepts, new proof procedures and new theories. This last domain of invention may not be routine, but it recurs periodically in the small and great advances within the discipline.

Mathematical terms, concepts and theories can never shuck off the values involved in setting them up, and that continue to reside in their use and development, however unanticipated these developments may be. For they are constructed or chosen to embody their originators' values during the pursuit of certain goals and interests. Even in subsequent usage, the terms, concepts and objects bear the shaping and directedness that reflects their origins. Every use of mathematical objects, be they terms, concepts, methods or theories, follows from acts of choice and valuation. Most notable in the present context, some of the values that permeate mathematics are ethical values. I have already mentioned openness and democracy as two such values. Indeed it is claimed that it was democratic argument in the first western democracy, Ancient Greece, that gave rise to logic (Lane 2018) and subsequently to mathematical reasoning and proof (Kleiner 2012). So it can be claimed that proof not only has an open and democratic function, but that it is itself a by-product of democracy and as such reflects its origins. Mathematics also entails, at least in part, the values of truth, honesty, fairness and justice. The practice of mathematics focuses on the pursuit of truth within its chosen domains, primarily by means of proof and argument. The pursuit of truth and the rejection of falsehood constitutes honesty, and any attempts to subvert truth dishonestly are eliminated by the open democratic scrutiny procedures intrinsic to mathematics. Fairness and justice arise from the explicit and justified reasoning central to mathematics, in which neither fear nor favour, preference nor prejudice can sway the logical outcomes.

Justification, which is central in mathematics, has the same roots as the term 'justice'. From the fourteenth century on justification has meant the action of justifying and the administration of justice, and justice is the quality of being fair and just; the exercise of authority in vindication of what is right (Harper n.d.). Justice depends on the open justification of decisions which is the basis of both mathematics and democracy. Mathematics, like democracy, is fair because of this openness and potentially equal treatment of all with respect to knowledge claims; their warranting and decisions as to their status as knowledge. This is not just a modern development. In the ancient societies of Mesopotamia and Egypt, where the discipline of mathematics was invented, the reliability of calculation, measures and numerical records was understood as part of the idea of justice, taking on an ethical value (Høyrup 1994). The openness and democratic checkabilty of accounting, taxation and trade calculations enabled them to be trusted and relied upon by all parties involved.

Purism

Associated with traditional objectivist or absolutist philosophies of pure mathematics is the ideology of purity, often termed 'Purism' (Restivo 1985). There are two meanings to the term 'purity'. First, there is its purely descriptive sense. Second there is the normative or prescriptive sense. Descriptively, the first sense of purity applies to

something that is unmixed, undiluted or is in its basic or primal state, be it a substance or practice conducted solely for its own sake. The second normative, prescriptive or evaluative sense of purity applies to something that is unadulterated, unpolluted or unbesmirched. In this sense to describe something as impure is to say that it is tainted. It has been degraded either aesthetically (less beautiful, more ugly) or morally (less good, more bad). Even when purity is used in the descriptive sense, the ambiguity of the term risks smuggling in the gratuitous negative evaluation of impurity, as a connotation. But often the term 'purity' is used by purists in its full, double barrelled sense, both descriptively and normatively.

Although the adjective 'pure' in pure mathematics is descriptive, Purism in mathematics is sometimes based on the normative or evaluative sense of purity and as such is a position concerned to keep out what are perceived as impurities and taint. Thus any signs of human presence, interests or values are excluded as detracting from the objectivism and absolutism of mathematics. Any attempt to attribute human interests or values to pure mathematics itself, not just to the practices and human foibles of mathematicians, is antithetical to the pristine purity of the subject. Values including those of ethics are repudiated by purists whose concern is to keep out the taint of ulterior motives such as utility and applicability, and to defend pure mathematics as intrinsically valuable and self-driven; knowledge pursued and derived purely for its own sake. As I argue subsequently, this chimes with MacIntyre's notions of the goods internal to a social practice.

Kline (1980, p. 303) notes the tendency to Purism in modern mathematics and indeed is critical of it.

> Mathematicians no longer hesitate to speak freely of their interest solely in mathematics proper and their indifference to science. Though no precise statistics are available, about ninety percent of the mathematicians active today are ignorant of science and are quite content to remain in that blissful state. Despite the history and some opposition, the trend to abstraction, to generalization for the sake of generalization, and to the pursuit of arbitrarily chosen problems has continued.

A well known proponent of Purism, and one of the few to explicitly articulate this philosophy is the renowned number theorist of the early twentieth century, Hardy (1940), who wrote the following.

> [A] real mathematician has his conscience clear; there is nothing to be set against any value his work may have; real mathematics is […] a "harmless and innocent" occupation. (p. 44)
> I have never done anything "useful." No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the world. (p. 49)

Here Hardy extols the lack of utility of pure mathematics and its ethical neutrality. Implicit in his praise is derogation of applied or impure mathematics. Part of the ideology of Purism is that the pure is beautiful, the impure ugly.

> Beauty is the first test: there is no permanent place in the world for ugly mathematics. (op cit, p. 14)

My interpretation of Hardy is that he subscribes to both the positive and negative tenets of the ideology of Purism in mathematics.

> *Positive tenet of Purism*: Pure mathematics is the superior form of mathematics: it is virtuous, beautiful and intrinsically valuable, standing in no need of justification. It is the product and expression of the virtuosity of the excellent mathematician.

> *Negative tenet of Purism*: Impure or applied mathematics may be a necessary evil for practical purposes but it is an inferior form of mathematics driven by extrinsic and ulterior motivations.

Purity is an important value within mathematics. Ancient Greek thinkers, including Plato were the first to distinguish pure and applied mathematics and to exhibit purist values. Pure mathematics was said to concern 'true being', the realm of pure thought concerning ideal objects and relationships, and thus was seen to be of supreme value (Plato 1941). In contrast, applications of mathematics were regarded as lowly activities performed by lesser beings for mundane practical purposes. Indeed in Ancient Greek society mathematics and philosophy were the preserve of the leisure class of free citizens, whereas applications of mathematics were the domain of tradesmen and slaves.

In the modern era, calculation and practical mathematics have also been given low status and viewed as mathematically trivial and philosophically uninteresting. It has been said that 'proper' or 'real' mathematics began around 2500 years ago in Ancient Greece, with the invention of proof as we now know it, signalling the birth of pure mathematics. From this perspective, the preceding 2500 years in which numeration, calculation, and measurement were invented and systematised are discounted.

> Oriental mathematics may be an interesting curiosity, but Greek mathematics is the real thing. (Hardy 1940, p. 12)

Purist values are reflected in the valuation of proof as a higher form of reasoning, and calculation as a lower form, from Plato onwards (Ernest 2009). Purism values pure proof-based mathematics as being significant epistemologically, and pertaining to truth, wisdom, high-mindedness and the transcendent dimensions of being. Equally this ideology denigrates applied mathematics and calculation as technical and mechanical, pertaining to the utilitarian, practical, applied, and mundane; understood as the lowly dimensions of existence. Pure mathematics was the domain of philosophers and free thinkers. Applied mathematics (termed logistic) was the domain of slaves and tradespersons.

However, purity has only been a dominant value for a fraction of the whole history of mathematics. It was powerful in ancient Greek times and in modern times, from the nineteenth century onwards.

> 'Applied mathematics' is an 'insult directed by those who consider themselves 'pure' mathematicians at those whom they take for impure (Truesdell, quoted in Kline 1980, p. 302)

An important question therefore is why does Purism come to dominate during these eras, when it was not seen as important for say, Newton, in the seventeenth century? Kline (1980) documents the growth of Purism from the nineteenth century onwards in a chapter on the isolation of mathematics, Chapter XIII: The isolation of mathematics.

A full historical and philosophical analysis of Purism in mathematics would require a book length treatise.[5] But a number of possible reasons for the growth of Purism in mathematics are proposed here.[6]

1. An absolutist or Platonist philosophy of mathematics promotes an idealised version of mathematical knowledge that is pure, superhuman and values-free. This philosophy supports Purism (Erdős in Wikipedia 2020; Hardy 1929; Penrose 1989; Balaguer 1998). However, alone it is not enough to generate the ideology of Purism. This philosophy is at best necessary, but not sufficient, for Purism to flourish in the narrow sense that claims pure mathematics is superior to applied mathematics. Thus, the Neo-Platonist outlook that was widely held in the early scientific era of Galileo, Descartes, Newton, etc., regarded all academic knowledge including science idealistically, as belonging in a realm far above and superior to mundane earthly matters. Physics and the associated applied mathematics were held in the same high regard as pure mathematics.
2. The standard presentation of pure mathematics in the modern era follows the Euclidean model and sequence, including, in order, definitions, axioms, lemmas, theorems. Lakatos (1976) argues that this order is an inversion of the historical development of knowledge. Such an inverse presentation conceals the complex historical process of conjectures, proofs and refutations, with all of the concept redefinitions and rule adjustments involved that reveal human agency and invention at work. Thus the standard presentation model represents a purification of proofs that serves to smooth out and eradicate marks of active human choices and labour in construction of mathematics, thus reinforcing Purism, for logic alone shines through.
3. British education until the nineteenth century was dominated by the Classics. Many of those who went on to become active in mathematical research would have been well versed in the Greek Classics including Plato and Euclid and would most likely have been influenced by their idealist and purist tendencies. Students of Euclid will have learned straightedge and compass geometric constructions, and will have learned that graduated rules and measuring devices are the inferior and imprecise tools of the engineer, and are disallowed in geometry which concerns only pure reasoning and proof.
4. In a hierarchical class divided society in which mathematicians are part of a higher class there is an ideological differentiation between the intellectual work of the elite and the manual or practical work of the lower classes. This is true among the nineteenth century and early twentieth century university dons, such as Hardy,

[5] Note that there is another form of purity applied within mathematics, which is not that discussed in this paper. This is a property of a proof (Detlefsen and Arana 2011). "Roughly, a proof of a theorem, is "pure" if it draws *only* on what is "close" or "intrinsic" to that theorem." (Arana 2017, p. 201).

[6] Cultural anticipations and supports for purism may be attributed to the Christian separation of the sinful and tainted world of the flesh from the pure and soaring realm of the spirit, and even to Cartesian dualism that contrasts the pure realm of reason with the mundane and mechanical world of the body.

just as it was among the Greek thinkers. This division helps to position class-linked knowledge and practices in a hierarchy, with pure mathematics positioned as superior to its applications, putting pure theory above applied practice. The underlying class ideology provides support for Purism.

5. The professionalisation of university teaching and research in the early to mid-nineteenth century at first in Germany led to boundary maintenance between professional mathematicians and outsiders and others with an interest in mathematics. More generally, it resulted in the demarcation of knowledge into disciplines reflecting university departments, and helped to reinforce Purism (Ferreirós 2016). The maintenance of boundaries between subject areas while serving to defend the pursuit of knowledge for its own sake from outside interests also maintains the purity of the disciplines (Restivo 1985). Hacking (2014) adds that the resurgence of Purism in early 19th century Germany can be traced directly back to the influence of Kant and his imposition of the categories of pure and applied reason.
6. The pursuit of any art for art's sake shrugs off applications, utility, or social responsibility as at best irrelevant distractions and at worst enemies of the art itself. This is one of the markers of decadence and Purism, seen across all the arts in the late Victorian era, as well as in the modern era. This ideology chimes with and may well help to reinforce Purism in mathematics.
7. The growth of interest in the foundations of mathematics with major work by Cauchy, Weierstrass, Cantor, Dedekind, Frege, Peano, (Hilbert), Russell, Whitehead and others turned the focus of many leading mathematicians inwards, or reflected their already inward gaze, into the depths of pure mathematics and away from the world and applied mathematics. This reinforces the ideology of Purism. I bracket Hilbert in this list because despite his interest in foundations he never turned his back on applications. Ferreirós (2016) documents not only how the earlier foundational work in Germany both helped to create and sustain a purist tradition in professional mathematics, but also how this continued in the early twentieth century work on logical foundations.
8. During the past 200 years or so, Ancient Greece has been 'talked up' as the starting point of modern European thought, and the 'Afroasiatic roots of Classical Civilisation' have been neglected, discarded and denied (Bernal 1987). Alongside and intellectually justifying European empire building and conquests in the Southern, Western and Eastern continents there has been a growing Eurocentrism, the racist bias that claims that the European 'mind' and its cultural products are superior to those of other peoples and races. Against this backdrop it is not surprising that that mathematics has been seen as the product of European mathematicians. However, there is now a widespread literature supporting the thesis that mathematics has been misrepresented in a Eurocentric way, including Almeida and Joseph (2004), Joseph (2000), Powell and Frankenstein (1997) and Pearce (undated). A common feature of Eurocentric histories is to claim that mathematics was primarily the invention of the ancient Greeks, with its foregrounding of pure geometric proof. The Indian contributions of zero and numerical expansions of infinite series are downplayed as are the Arabic developments in algebra (Joseph 2000). For example, according to Pierre Duhem "Arabic Science only reproduced the teachings received from Greek science" (Rashed 1994, p. 338). Eurocentrism has promoted

Purism across the whole of European intellectual endeavour, especially in the history and philosophy of mathematics.[7]

9. Many of these influences are historical, stemming from the late nineteenth to the mid twentieth century. However, mathematicians are taught and shaped as professionals by working with an older generation of established mathematicians, whose culture, including values and ideologies as well as their knowledge and methods, are passed on from generation to generation (Kitcher 1984). Not only are such cultures and values passed on to new mathematicians, but they are also passed on to many high school mathematics teachers whose preparation is normally conducted in part by mathematicians. Thus the Purism of the past is transmitted forward and thus partly preserved among mathematicians and teachers in modern times.

For reasons such as these Purism remains a prominent value for many mathematicians up to and including the present day. Purism rejects any human features or influences on pure mathematics. Purism actively protects the disciplinary boundaries against any breaches from the taint of traces of human activity or values. Thus from this perspective ethics is perceived not only to have no bearing on pure mathematics, but any suggestion that it might have is repudiated as a threat to its purity.

A problem for relating ethics to mathematics is that from the perspective of Purism mathematics is regarded as neutral and value free. Ethics is regarded as the set of values most irrelevant to mathematics. Any attempts to raise ethical issues with regard to pure mathematics are seen as possibly tainting or lowering the subject from its elevated state of purity. Purism strenuously rejects any suggestion of the relevance of ethics to pure mathematics, and usually does so without serious consideration of ethics. On the face of it, to say that one mathematical concept is more vicious or virtuous than another seems absurd.[8]

However, MacIntyre's virtue ethics provides a way of respecting the ideology of mathematical Purism, at least in significant part. According to his theory the social practice of pure mathematics should focus exclusively on the internal goods of the practice. A virtuous pure mathematician will strive for these internal goods and eschew the external goods of pure mathematics, including uses, applications, personal rewards and fame. Thus MacIntyre's theory accommodates mathematical Purism, as well as other ideologies of pure mathematics, by focussing on the internal goods of the social practice. A benefit of this is that a lack of focus on purely internal goals within a mathematical practice could lead to a decline in the practice itself.

There is some evidence that the strong demarcation between social practices of pure and applied mathematics is weakening. First of all, several topics that were exclusively pure, like number theory, have major and highly visible applications. Number theory

[7] Simultaneous with the growth of Eurocentrism has been that of Orientalism, a fascination with the arts and cultures of the East. But these have been seen more as exotic, titillating and even as objects of contempt rather than as an equal partner with European culture (Said 1978). I'm unaware of it affecting views of mathematics in any ways that redress the imbalance of Eurocentrism.

[8] Johnson (2012, 2017) argues that that the formulation of probabilistic concepts and market equations has important and different ethical consequences for society, and MacKenzie (1981) shows how different statistical concepts by Pearson and Yule had profound effects on eugenics programmes, so the ethical evaluation of mathematical concepts is not, in fact, absurd.

is of course deeply involved in cryptography, digital representation, digital computing and all of information and communication technologies. Computers are a standard tool for pure mathematicians and have led to the growth of non-traditional and quasi-empirical proofs that straddle the pure and applied border (Tymoczko 1986). Secondly, even in modern times many professional mathematicians do not fully subscribe to the purist ideology. Grigutsch and Törner (1998) investigated the views of mathematics of 119 university mathematicians in Germany. They found that more mathematicians viewed mathematics as process-based problem solving and applied; concerned with practical use and relevance to society; than viewed it as purely formalist or Platonic concerned with "aesthetic divine games". This suggests that purity was not the dominant value present.

In contrast, Müller (2018) asked Cambridge University students to characterise pure mathematics and identified four typical responses which he used as overall categories for the analysis of his data.

1. Pure mathematics is a pursuit for its beauty.
2. "I do it because it is completely detached from reality."
3. Mathematics is studying the mind of God.
4. The puzzler enjoys the buzz of solving a perfectly-defined problem or puzzle within the realm of mathematics.

Each of these perceptions reflect a purist ideology, to a greater and lesser extent, and make little room for a consideration of ethics within mathematics because of the detached, inward focus. Given the recency of his study the claim that Purism in mathematics is disappearing would seem to be an overstatement.

Ethics and applied mathematics

Does MacIntyre's theory also apply to applied mathematics? Pure and applied mathe- matics can be regarded as different social practices. In the past the boundary between them was maintained by a number of factors including the following:

1. The separation between the university departments of pure mathematics and applied mathematics, not to mention statistics and computing,
2. The different sets of publications and problematiques of the cultures of pure and applied mathematics,
3. The different ideologies and philosophies of pure and applied mathematicians (especially the Purism of pure mathematicians).

The social practice of applied mathematics has its own internal goods, the goals of the virtuous applied mathematician. These are not the purist goals of the practice of pure mathematics since they encompass utility, usefulness and applicability both to real world problems and to those within other disciplines. These are internal goods because they look primarily to successful problem solutions within the social practice of applied mathematics and not to external goods like personal advancement, rewards or fame. Thus the virtuous applied mathematician will seek the goods internal to this social practice. These include the virtues that lead to good (well made) applications and 'real world' models with power, elegance, economy of form, generalisability and

effectiveness. Such products may exhibit a form of beauty visible to those with the insider knowledge needed to appreciate it, such as participants in the social practice. Like that of pure mathematics the social practice of applied mathematics is widely regarded as neutral and free from social responsibility. Although acknowledged to be value-laden, the values identified are utility, efficiency and so on. Applied mathematics is widely seen as merely technical knowledge, brought into solve scientific or real world problems that originate outside of mathematics. Like a bag of tools, mathematics is used in the service of externally posed problems, and applied mathematicians are responsible for the *means* of solving the problems, not the *ends*, that is, the nature and significance of the problems themselves. Thus in mathematics as a tool "Control replaces explanation, and validation is accomplished by use" (Lenhard and Carrier 2015, p. 18). Within applied mathematics applications are generally evaluated in terms of their efficacy and utility in the narrow context of the problem posed, and any issues concerning the problem's wider social significance are not only ignored but viewed as beyond the business of mathematicians. Their role is simply to provide technical support. The research of Chiodo and Vyas (2018, p. 2) discovered the widespread view that "while there are ethical issues in applied mathematics, these are imported from the disciplines that the mathematics in question is being applied to, and thus do not require a separate mention". It should be noted that these authors report but do not endorse this view, they are part of the movement to raise the profile of ethics in mathematics.

Although there are a growing number of exceptions, many applied mathematicians do not see themselves as having any social or ethical responsibility for their mathematical work. Chiodo and Bursill-Hall (2018, p. 5) summarise the beliefs of both pure and applied mathematicians as "Believing there is no ethics in mathematics - This is where most mathematicians are today." If an applied mathematician declines on ethical grounds to work on developing a missile guidance system or an App that invades people's privacy, it is seen as a personal choice. Likewise, a vegetarian might object to working in a slaughterhouse or meat packing plant on ethical grounds, but that is seen as a personal choice based on personal values rather than a society wide ethical imperative.

As a technical parallel, you would expect a mechanic to locate and fix a problem with your diesel car. However, if they took you to task for driving an environmentally damaging vehicle you might counter that this is none of their business and that they are overstepping the limits of their professional role, which is technical.[9]

It is informative to contrast the received views about the responsibilities of mathematics and mathematicians with parallel views about the social responsibilities of science and scientists. In Ernest (2018) I document the widespread if not universal agreement on the social responsibility of science. Resnik and Elliott (2015, p. 1) support their claim that "Numerous scientists and philosophers have argued that scientists have a responsibility to address the social implications of their research" with reference to 17 further publications, and the literature extends well beyond this. Thus many argue

[9] The recent and current impact of climate change activists, most notably Greta Thunberg, the Extinction Rebellion protesters and environmental movements may already have changed this.

that the Promethean power of modern science and technology warrants an extended ethic of social responsibility on the part of the scientists and technologists.

The situation is very different in mathematics. Very few voices are raised calling for an ethic of social responsibility of mathematics, pure or applied (Ernest 2018).[10] In science, technology and other areas of use across society, applied mathematics is seen as the servant, the under labourer whose job is to do the work, not to question the task. Thus perceptions of applied mathematics do not challenge the denial of ethical responsibility associated with pure mathematics. For different reasons, applied mathematics is also regarded as neutral and free from social responsibility, merely a set of tools used by others. It is a technical activity, judged solely on its utility and effectiveness.

MacIntyre's theory of ethics provides a way of maintaining the purity of pure mathematics, and the ethical neutrality of applied mathematics within an ethical system. Both social practices have internal goods that can be construed as the goals or targets of sets of virtues, but which exclude traditional ethical judgements about whether particular attainments of these targets are virtuous or vicious. Such judgements are reserved for the second and third levels of MacIntyre's theory.

Is this a strength or weakness of the theory? It is a strength in allowing mathemati- cians that focus exclusively on the internal goods of the social practices of mathematics to be judged good, excellent and virtuous. Restivo (1994) argues that Purism is an intellectual strategy serving social goals including the demarcation of knowledge and defending the pursuit of knowledge for its own sake from outside interests. This can be useful in protecting the interests of a discipline such as mathematics from political interests that might seek to bend it towards more immediately useful applications, as was witnessed in Soviet Russia under Stalin (Grossman 1980). If mathematicians were to be judged virtuous for including external goods as goals within their practices, including external ethical judgements about the virtue of some applications, then other external goods might also intrude, deflecting the social practices away from internal goods. Mathematical problems could be chosen for solution because of the rewards of fame their solutions bring, as with Cardano and others in the sixteenth century equation solving 'duels', rather than their import and significance for the mathemati- cal practices. What counts as good in mathematics is the mathematician's virtuosity, the virtues that give rise to their excellence as professional mathematicians working towards the inner goods of mathematical practices. Furthermore, I believe that the best judgements of what constitutes excellent advances in mathematics come from mathematicians themselves. Constraining them from pursuing the most intellectually exciting problems and most significant results in mathematics, as judged by them- selves, seriously risks stultifying their activities and the growth of the discipline.

However, the exclusion of traditional ethical judgements is also a weakness, in that mathematicians who eschew the second and third levels of MacIntyre's theory can regard themselves as virtuous as well as being ethically neutral at all levels in the mathematical practices. No matter how troubling the applications are, such mathe-

[10] Recently a project concerned with ethics in mathematics has emerged, based at Cambridge University, UK, with an international network of scholars and conferences on ethics in mathematics starting in 2018 (EiM 2018). There is also a growing literature on the ethics of computer applications of mathematics (for example, MacKenzie 2006 and Verbeek 2011).

maticians can absolve themselves of any ethical responsibility. This is despite the fact that no mathematical theories are entirely innocent, for as we have seen in the twenty-first century all are potentially applicable. Although following the internal dictates and norms of the social practice of mathematics cannot be equated with "just following orders", there remains an uncomfortable resonance between the two. Indeed, I have argued that the current training of mathematicians in a climate that eschews ethical considerations as irrelevant can lead to persons ill-suited to making social, political or ethical decisions, due to their having learned to compartmentalise and hive off such considerations from rational decision making (Ernest 2018). But it can still lead to mathematicians of great virtuosity within the social practice of mathematics itself.

However, mathematicians that adopt a fully a-ethical stance throughout will not be judged virtuous in terms of MacIntyre's theory, for one cannot pick and choose among the three levels. It is not clear that intensely focusing on research to the exclusion of everything else in one's life is a characteristic that could count as contributing to the successful pursuit of a unified human life or even the sustaining of the social practice of mathematics more generally. To overcome this potential ethical lacuna it is necessary to develop and stress the second and third levels of MacIntyre's theory as essential concomitants of ethical social practices in mathematics.

The second and third levels of MacIntyre's virtue ethics

The second level of MacIntyre's theory requires that to be virtuous an individual must have a personal narrative encompassing the social practices in which they participate, and indeed reflecting the whole of their chosen way of life. For MacIntyre (2007, p. 252) "the concept of an intelligible action is a more fundamental concept than that of an action." Thus a narrative makes sense of the actions in a life, through their intelligibility, and relates them to its *telos* or the purpose of that life. The narrative must also be consistent and consonant with the culture and the moral tradition of which they are a part.

According to MacIntyre we are all part of, and responsible to a greater tradition, necessarily including moral elements, and these make up the third level of his theory of ethics. He is careful to avoid the pitfall that comes from assuming that there is only one overarching moral tradition worldwide. Our moral accountability lies within our tradition, such as humanism or the secular Judeo-Christian tradition of the Anglophone West. In addition, we have the responsibility to monitor, develop and extend the moral tradition to which we belong. It is not a static and unchanging framework. MacIntyre draws on the thought of Thomas Kuhn and Imre Lakatos to define tradition as an element of an ongoing practice of inquiry, which might include large revisions of inherited theory and practice (Devine 2013).

> The virtues find their point and purpose not only in sustaining those relationships necessary if the variety of goods internal to practices are to be achieved and not only in sustaining the form of an individual life in which that individual may seek out his or her good as the good of his or her whole life, but also in sustaining those traditions which provide both practices and individual lives with their necessary historical context. (MacIntyre 2007, p. 219)

The second level of MacIntyre's theory posits that the excellent human agent has the moral qualities to seek what is good and best both in practices and in life as a whole. In expressing these qualities in our lives we have the duty to examine and evaluate the ethical implications and outcomes of our social practices, such as the social practice of mathematics. We do this in the light of our moral, intellectual and lived tradition, which, like our social practices, must be monitored and extended organically, with the agreement of our community, in order to better sustain the good of our practices and the good of society.

Thus to be a good mathematician is not only to be excellent or good enough within the social practice of mathematics. It is also to care for, and take responsibility for the impact that our mathematical practice and our overall lives have on society. As active ethical agents we need also to monitor the standards and actions of society as a whole within our tradition, moral and beyond, and to extend that tradition to better enhance the good of society, its practices and its citizens.

In addition, to be a good mathematician from the perspective of the second level of MacIntyre's theory is also to act ethically and virtuously within the social practice of mathematics: it is to be a *good* mathematician, that is to be a good person within the social practice of mathematics, beyond being a good *mathematician*, that is someone manifesting the virtues devoted to the internal goals of the social practice of mathematics. Naturally, the virtues involved cannot be specified completely or near completely as the virtues depend on the social practice, its cultural and historical context, and the tradition with which it sits. In addition, virtues are to a large extent tacit, manifested through virtuous behaviour rather than adherence to explicit norms or criteria. But one would expect the virtues to include helpfulness to colleagues and students, honesty toward colleagues, responsibility in sharing the work of the institution and practice, conscientiousness in conducting one's professional life, and so on.

Of course there can also be ethical conflicts between the first and second levels of MacIntyre's theory (and beyond). Extreme focus on obtaining the internal good of the practice of mathematics, namely mathematical work, at the expense of support for students and colleagues, neglect of teaching (unless that is already counted as a good of the practice), and poor behaviour towards colleagues and students, can be viewed as a failure at the second level. It is hard to imagine a personal narrative that will justify or extol such behaviour. However in the mythology of mathematical practice, great mathematicians that neglect other dimensions of virtue are tolerated because of their major contributions to the good of mathematics, namely, proving difficult theorems and establishing new theories. Ironically, these may be the same mathematicians that obtain goods external to the social practice of mathematics, namely fame, popularity and financial rewards. However, when thus attained, these external goods are secondary to and follow on from the attainment of the internal goods of the social practice of mathematics. So they should not detract from the dedicated pursuit of the internal goods.

In addition, paralleling the above imbalance but the other way around, excessive attention to students and teaching, or to institutional administration, despite being manifestations of a virtuous person and professional, if they are performed at the cost of pursuing the goods of the practice, namely crafting and creating mathematics, can be viewed as a failure within the primary social practice of mathematics.

One apparent problem in MacIntyre's theory is that the virtues we cultivate in order to be good enough or excellent agents in the social practice of mathematics should also enable our virtuous functioning at the second and third levels of his theory. What virtues can we propose that are manifested in a good mathematician's practices, as well as in a good person living a unified well narrated life and thirdly enabling virtuous contributions to our cultural traditions? To be applicable at all three levels these must be more general than mathematics-specific virtues such as precision in symbolism, fastidiousness about detail in proofs, an aesthetic appreciation of rationality, according proper due to others' priority claims, willingness to engage with rival understandings, and to expose one's own understanding to challenge, and willingness to share understandings with others (Corfield 2020).

> "Checkability" would not be a recognized virtue if someone like Serre (but there's no one quite like Serre) hadn't proposed it as a criterion. At this point a historian is needed to tell us whether the virtues of mathematical practice come and go according to the preferences of acknowledged leaders, or whether they show some constancy over different periods. (Harris 2020)

This brings me back to the problem I considered at the start of this paper. Is it possible to specify a set of virtues that a good mathematician, that is, someone pursuing the goals of the social practice of mathematics needs to possess at all three levels? A careful reading of MacIntyre's theory suggests that this is the wrong question to ask. For although contemporary virtue ethics proposes an alternative to modern moral theory, it takes for granted that the purpose of ethics is to provide a moral epistemology. Contemporary virtue ethics purports to let agents know what qualities human beings ought to have, and the reasons that we ought to have them, not in terms of our fitness for human agency, but in universal, disinterested, non-teleological terms. MacIntyre's ethical project examines the virtues in this way, but it is not a branch of moral epistemology.

> For MacIntyre, moral knowledge remains a "knowing how" rather than a "knowing that;" MacIntyre seeks to identify those moral and intellectual excellences that make human beings more effective in our pursuit of the human good. MacIntyre's purpose in his ethics of human agency is to consider what it means to seek one's good, what it takes to pursue one's good, and what kind of a person one must become if one wants to pursue that good effectively as a human agent. (Lutz n. d., Section 5a).

As a mathematician, or indeed as a person, seeking one's good is a process of living a self-reflective life; clarifying one's goals as well as the means one is adopting in striving to achieve them. The reflective practitioner living the examined life rather transcends or exceeds any specific list of virtues since it is a 'knowing how' type of wisdom rather than a 'knowing that' kind of knowledge. So the fruitless task of trying to spell the virtues out can be laid to rest.

3 The ethics of mathematics

Using MacIntyre's virtue theory as a moral yardstick suggests two dimensions to the ethical or good mathematician. First, the mathematician needs to cultivate those virtues that enable the attainment of the goods internal to the social practice of mathematics. These are both epistemic and moral virtues since they lead to good mathematics, as judged by the community of mathematicians. Second, the mathematician needs to be aware of and take some responsibility for how the social practices of mathematics to which she contributes contribute to and impinge on the overall good of society. What this good is, and how it is defined, are part and parcel of the encompassing moral and cultural traditions of the society.

In consequence of this position, the pure mathematician who seeks for and justifies her mathematical work only through the goods of the social practice of pure mathematics is justified at the first level. The applied mathematician who only judges the success of her modelling and applied activities through their utility and effectiveness in describing and predicting their targeted outcomes is also so justified. Both mathematicians are primarily addressing the goods internal to their practices. But as virtuous mathematicians they need also to stand back and take a second-order look at the outcomes and the impact of their social practices on society and respond to what they see. As human beings and citizens in society mathematicians are not absolved from the ethical consequences of their work. They have a dual responsibility, to be virtuous mathematicians in the narrow sense restricted to the goods of the social practice of mathematics, and to be virtuous contributing and responsible persons, in the wider sense in which we are all contributing members of society.

So much for the virtuous and ethical mathematician, but how does this impact on the ethics of mathematics? Some authors are fond of distinguishing between what is termed Mathematics and mathematics (Bishop 1988). Mathematics with a capital *M* is the sum or body of formal mathematical knowledge that exists independently of mathematicians. The lower case *m* in the term mathematics signifies that it comprises human mathematical practice(s). If this distinction is valid, then defining the ethical mathematician only pertains to mathematics, and not to Mathematics. It can be argued that Mathematics as abstract and objective knowledge is beyond good and evil; that is, outside of what can be termed ethical. This may be the position of purists and absolutists. It is a defensible position, for if ethical action can only be undertaken by a moral agent, and given that Mathematics is abstract and not agentic, then a fortiori it cannot be an ethical agent. This is a legitimate conclusion, but it does not necessarily entail that Mathematics is free from ethical values.

My position, and that of social constructivist and humanistic philosophers of math- ematics, is that Mathematics per se does not exist outside of social practices. There is no such abstract entity in the world or elsewhere. This is an ontological position, for we claim that only the material world exists, and within it there are social practices and their participants and products. Mathematics only exists as a complex product and part of social practices, thus in definite material form there is only (lower case *m*) mathematics. "Capital-M Mathematics is a purely hypothetical subject invented by philosophers to address (for example) problems of truth and reference" (Harris 2015, p. 30).

However, Mathematics (upper case *M*) does exist as a cultural entity, but it is far from distinctive and unique and it shifts in its meaning and reference across different cultural groups, social practices and individuals, as well as historically. Mathematics (upper case *M*) in its varying forms is value laden, just like any other product of the human hand or brain, but as I have argued, cannot be an ethical agent.

What is ethical, and is open to ethical evaluation, is the set of mathematical practices and applications of mathematics across society and the world. In Ernest (2019) I examine the impact of mathematics in education and throughout society from an ethical perspective. In conducting an ethical audit I consider four aspects of mathematics. These far from exhaust the ethical dimensions of mathematics that one might consider. The first two pertain to the role of mathematics in society via education. The next two focus on the impacts of applications of mathematics. I outline them briefly here.

First of all, there is the high, and I argue exaggerated, social valuation of mathematics, and the concomitant impact this has on society, especially via education. Mathematics is very highly valued in society because of its undeniably great and widespread utility. However, there is a fallacious inference that because of this great utility, and because the mathematical needs of society are also great, therefore all students must be taught mathematics to the highest possible level.

This raises the question, of what are the actual mathematical needs of society? It is useful to apply Marx's fundamental distinction between use and exchange value. In education, the difference between use value (actual utility) and exchange value (social or opportunity value) of learning is the educational and social advantage or obstacle that is afforded by needing mathematical certification of learning. My claim is that the use-value of mathematical certification for everybody across society is less than is widely claimed and the exchange-value is exaggerated.

My claim is that everyone needs 'numeracy plus' to be functioning critical citizens in a modern democratic society. They need to have mastery of the mathematics underlying their everyday lives including consumer and economic decisions. As functioning modern citizens, they need to be able to interpret and evaluate the uses of mathematics in social, commercial and political claims in published reports, newspaper and other media presentations, advertisements, financial documents, and so on. By 'numeracy plus' I mean the content of elementary school mathematics plus some additional knowledge, such as understanding and skill in using and interpreting data representation and processing, spreadsheets and elementary algebra, probability and statistics, ratio and proportion, reasoning and practical problem solving. This should include understanding algorithms, Apps, and big data in principle but not necessarily in detail. Such knowledge needs to empower elementary and everyday applications, rather than being directed exclusively at completing written tasks in external examinations and assessments at 16 years or thereafter (Ernest 2019).

Although every youth and adult needs 'numeracy plus' in order to be fulfilled and economically functioning critical citizens in a modern democratic society, a much smaller group need (or want) higher or advanced mathematics. I argue that because of the ubiquity and importance of mathematical applications there is an overvaluation of mathematics that distorts education for all. The needs of the few are generalised, resulting in a demand for high stakes test certification from the many. Such mathematics testing operates as a critical filter for entry to further study and almost all

professions, and this is class reproductive in distributing social advantage along the lines of social capital. Although this is a social rather than mathematical problem, mathematicians cannot simply look the other way. Mathematicians and mathematics-related professionals, such as myself, are complicit and undoubtedly gain from the overvaluation of mathematics in society.

To claim that most people do not need much advanced mathematics at the same time as society is becoming increasingly mathematised, may seem like a contradiction. But it is not intended to be so. What I term 'numeracy plus' represents a level of mathematical competency and confidence in its use that is probably beyond what 80% of the population have today. In 2011 a numeracy test taken by a sample of working age adults revealed that only 22% of the English population achieved results comparable to grade 3 or above at GCSE mathematics. This is widely regarded as the level of a pass, and even this guarantees little in the way of applicable understanding (National Numeracy 2012). Citizens competent to the 'numeracy plus' level should be able to understand, in principle, many Apps, applications and the widespread public uses of mathematics made for informational, political and commercial purposes. Such understanding will be in terms of functionality, without knowing about the detailed internal equations and models within applications. Thus it represents a higher level of comprehension and applicability than that which some GCSE level students gain at 16 years of age from their academic and examinations focussed studies of mathematics, not to mention the 78% of adults that do not achieve this level of certification.[11]

The number of citizens able to write or fully understand the mathematical models in play across the public domain, not to mention those in professional use, currently represents a tiny fraction of the population. I am all for increasing this by opening up opportunities in education for those wishing to pursue mathematics to GCE 'A' level, normally at 18 years of age, to undergraduate level, and beyond. It might also be worthwhile to add apprenticeships in mathematics, data, and information and communication technology management to current educational offerings. The net outcome should be increased levels of mathematical understanding beyond 'Numeracy Plus', at least concerning mathematical applications and applied mathematics throughout the populace, so there should be decreased risk of our becoming slaves to what we might term our mathematical masters.

Second, there also is the personal impact of mathematical studies on individuals that is neither happy nor good for all. Some learners are wounded by their encounter with mathematics and carry negative attitudes with them when they leave schooling. This reduces their mathematical functioning in society as a whole, as well as potentially reducing their career options and life chances. Teachers of mathematics should be cognisant of this, and share in the responsibility for it, alongside the educational systems and policies that are enacted in the imposed mathematics curriculum, pedagogy and assessment. I support this claim more widely elsewhere (Ernest 2019) where I also

make the case that successful mastery of mathematics risks developing instrumental thinking, unless ethics is included alongside the advanced study of mathematics.[12]

Third, there are ethical implications of the explicit applications of mathematics in society. Many examples could be cited as problematic, and not so long ago the huge personal data capture and exploitation affair surrounding Facebook and Cambridge Analytica was in the news (Cadwalladr 2017). Responsibility should be shared among those involved, including applied mathematicians, information and communication technology professionals, computing specialists and authors of algorithms and Apps, as well as those who fund, instigate and control these applications across society at great social costs, for political and economic gain. Expansions on this theme can easily fill a book (see, e.g., O'Neil 2016).

Fourth, there are hidden and implicit applications of mathematics in society that also have major ethical implications. The hidden use of programs to ascertain if persons are worthy of loans or credit, or are likely to commit crimes, can change those persons' lives without their knowledge or right of challenge. These and many more hidden control systems are currently at work, used by corporations and government bodies. In Ernest (2019) I focus on the performativity of the mathematisation of society as an ethical issue. For example, the widespread official and social identification of intelligence with IQ score not only limits the recognition of human capabilities, but also demonstrably impacts on policies, lives and human identities. Other measures such as evaluative scores in education can change individuals, schools, and national and regional education systems (Gorur 2016). There are troubling but often hidden applications of mathematics needing ethical scrutiny and oversight right across the worlds of governance, banking and financial markets. These are performative in shaping and driving policies and markets, and go well beyond being simply neutral tools (Johnson 2017; MacKenzie 2006).

The mathematisation of society is performative. Metrics and measures do far more than capture the concepts and qualities they are supposed to represent. They noticeably alter the social processes that they are implicated in, often with significant impacts, and these changes can have major negative ethical effects. When measures are imposed by institutions and policies to manage, shape and to evaluate social practices, the measures become instruments of power to change the practice; they redirect it (Power 1999). Thus, for example, a hospital, prison or school managed solely by the attainment of target measures of performance is at risk of compromising and dispensing with the care and professionalism traditionally provided by the key staff involved in the service. In their performativity, the forces of measurement and mathematics are radically reshaping and restructuring human practices and social reality, and even the futures and possibilities we can imagine. Beer (2016: p. 6, original italics) describes this process of performativity and its outcomes succinctly as the "relations that exist between *measurement*, *circulation*, and *possibility*." Mathematicians have a special duty to

[11] Will we see claims of PCSD (Post Cognitive Shock Syndrome), the lasting cognitive and affective damage inflicted on students unsuccessful in areas of mandatory school study such as mathematics? Although this idea is intended semi-humorously, the fact is that I frequently meet persons wounded by their encounter with school mathematics, and it is evident that aspects of their functioning in society are permanently inhibited. Currently considerable attention is being given to the major impacts of what are now termed Adverse Childhood Experiences, such as in Metzler et al. (2017).

acknowledge and identify these applications of their discipline and to scrutinise them ethically. As democratic citizens we share with all other citizens the duty to take responsibility for the direction that social policy is taking, and be cognizant of its social justice and ethical implications. ‘

The responsibilities of mathematicians

So what are the ethical responsibilities of the virtuous mathematician? I have suggested four dimensions of mathematical applications that call out for ethical scrutiny, two concerning education and two about direct applications in society. These four aspects are interconnected and may overlap to some extent. Furthermore, there are doubtless further domains that could be included, for example, I have barely discussed the ethical responsibilities of teachers of mathematics nor those of mathematicians as teachers in colleges and universities.

However, there is a deeper obstacle in play. In sketching these footprints of mathematics in society I am entering into a further social practice beyond the immediate practices of mathematics. I am working within the social practices of critical theory and critical mathematics education, verging on the philosophy of mathematics. The internal goods of these practices include constructing incisive critical and theoretic accounts of their target areas, notably mathematics and its role in education and society. Furthermore, I have chosen to focus on what I perceive to be the ethical shortcomings of mathematical applications in society. There is no moral imperative for the virtuous mathematician to accept my classification, nor to accept my ethical critique wholesale, especially in the undeveloped state in which it is presented here. The details of my ethical critique are largely independent of MacIntyre’s virtue ethics, and constructed without explicit reference to it.

What MacIntyre’s theory offers is a space to make an ethical evaluation of mathematicians’ activities within the social practices of mathematics, and beyond, without determining the form or content that such an evaluation should take. MacIntyre’s virtue ethics suggests that the virtuous mathematician has the moral qualities to seek what is good and best both in mathematical practices and in life as a whole. Thus the virtuous mathematician will strive to achieve the goods internal to the social practices of mathematics; seeking to create new and valued mathematical knowledge or effective models and applications of mathematics, according to the type of social practice in which she is engaged. In addition, virtuous mathematicians have the duty to examine and evaluate the ethical implications and outcomes of our social practices, including those of which they are most knowledgeable, the social practices of mathematics. This includes taking responsibility for the impact that mathematical practices and mathematicians overall lives have on society. Mathematicians have the responsibility to do this in the light of our shared moral and intellectual tradition. But they must make up their own minds about which outcomes are ethical, and which fall within the orbit of their moral responsibility. I have no warrant other than reasoned argument to impose my ethical critique on mathematicians. Some may find my critique, more fully detailed in Ernest (2018, 2019), too strong, too radical, and perhaps too political. However, MacIntyre’s theory far from imposing a ready-made solution, offers a space for virtu-

ous mathematicians to reflect on the inner goods of the social practice of mathematics, to reflect on the virtues that enable them to seek to attain these goals, and to seek an overall consistency in the virtues they manifest at all three levels, in an ongoing cycle of self-reflection and living an examined life.

> The virtues therefore are to be understood as those dispositions which will not only sustain practices and enable us to achieve the goods internal to practices, but which will also sustain us in the relevant kind of quest for the good, by enabling us to overcome the harms, dangers, temptations, and distractions which we encounter, and which will furnish us with increasing self-knowledge and increasing knowledge of the good. (MacIntyre 2007, p. 219).

Virtuous mathematicians cannot in all conscience turn their back on all ethical considerations. Even the purest of pure mathematicians, as citizens, must consider the ethical impact of their social practice on society and the flourishing of its members. They have the responsibility for developing their own ethical framework and judging the impacts of the social practices of mathematics, and the applications of mathematics, for good or ill, themselves. Scientists have long taken social responsibility for the impacts of science and its technological applications on society. Mathematicians are the last group of science professionals to acknowledge the social and ethical implications of their social practices, perhaps because of the widespread doctrines of Purism and neutrality.

MacIntyre's virtue ethics provides a moral theory that happily encompasses mathematics, mathematicians and their social practices. The widely valued purity or neutrality of mathematics is respected by MacIntyre's theory's insistence that the first focus of ethics be on the internal goods of mathematical practices. However, the next stage of his theory requires that the overall good life and the good of society must be the concern of the virtuous citizen, woven together to form the narrative order of a single human life. Beyond excellence, or being good enough, in the social practice of mathematics a good mathematician must also monitor and respond to developments in society as a whole from the perspective of our shared cultural and moral tradition, especially the impact of the social practices of mathematics. Mathematicians, including pure mathematicians, share the responsibility with all citizens for evaluating the ethical impacts of all applications of mathematics and to contribute to the overall goods of society as well as reflecting on and contributing to the social traditions in which we all take part.

I suppose it could be said that I am trying to square the circle. My argument, drawing on MacIntyre (2007) is that pure mathematicians should be free to pursue the subject for its own sake. The development of pure mathematical concepts, methods and theories extends and adds to the beauty and depth of human knowledge. It is intrinsically valuable like any other area of creative work including the arts. Limiting mathematical development to what someone else from outside of mathematics thinks is worthwhile, valuable, fruitful or even ethical puts undue constraints on the growth of pure mathematics. Similarly, deciding what art can be shown, or what novels and books can be

published limits the reach of literature and creativity in writing.[13] Furthermore, timeand again history has shown that 'blue sky' pure research in mathematics, extending beautiful theories and structures for their own sake, delivers knowledge that humankind has found inestimably valuable in areas beyond mathematics. So the argument that free and unfettered research wastes resources does not hold, although I am arguing here for the intrinsic value of mathematical knowledge, not for its extrinsic utility.

Because pure mathematics is developed with no thought of applications in science, society and the world, I am not uncomfortable with claiming that not even ethical considerations, and I can't imagine what these might be, should limit its growth. At the same time, I am arguing that mathematicians should accept more social respon- sibility for their work, and that society should reconsider how it views mathematics.Although the same virtues must permeate all three levels of MacIntyre's ethical theory, being a good *mathematician* means you should pursue the inner goods of the social practice of mathematics without the distractions or seductions of its external goods. This may mean ignoring any criticism or endorsement that comes from outside of the social practice of mathematics while researching pure mathematics. But as a *good* mathematician, that is, as a virtuous person, one needs to develop an integrated ethical narrative of the self. This must provide a balanced reflection on one's professional roles and activities within the totality of one's actions, including a deliberative aware- ness of how one is positioned in, and contributes to, one's cultural tradition. In this way I believe one can be both a good *mathematician* and an *ethical* mathematician.

Let me conclude by pointing to one outstanding example of such a person, namely Bertrand Russell. First of all, he was an outstanding pure mathematician and math- ematical logician, and certainly a purist in that respect. But second, he also devoted himself to peace and the improvement of the human condition. He was co-author of the Russell–Einstein Manifesto calling for scientists to take ethical responsibility forthe applications of their work (Russell and Einstein 1955). Third, he attended to the narrative of his life and wrote a three volume autobiography as well as numerous other insightful and reflective pieces. For all his personal weaknesses, which we learn of through his own disclosures, he was both a good, nay great, *mathematician* and an *ethical* mathematician.

Conclusion:

In this paper, we have explored the application of MacIntyre's ethics of virtue to the field of mathematics, with a particular focus on the concept of purism. We have argued that while the pursuit of mathematical perfection can lead to important discoveries, it can also create ethical dilemmas when it conflicts with other values and virtues. Applying MacIntyre's ethics of virtue to mathematics can lead to a more fulfilling and meaningful practice of mathematics by cultivating virtues such as honesty, courage, humility, and justice.

However, we have also acknowledged that applying MacIntyre's ethics of virtue to mathematics poses certain challenges and limitations, and that additional ethical frameworks or approaches may be necessary to address the complex ethical issues that arise in mathematics. Nonetheless, we believe that the application of MacIntyre's ethics of virtue to mathematics is a promising avenue for exploring the ethical dimensions of mathematical practice and cultivating a more virtuous approach to mathematics.

References

Almeida, D. F., & Joseph, G. G. (2004). Eurocentrism in the history of mathematics: The case of the Kerala School. *Race & Class, 45*(4), 5–59.

Arana, A. (2017). On the alleged simplicity of impure proof. In R. Kossak & P. Ording (Eds.), *Simplicity: Ideals of practice in mathematics and the arts* (pp. 207–226). Switzerland: Springer.

ASCL - Association of School and College Leaders. (2019). *The forgotten third, final report of the commission of inquiry*. Oxford: Oxford University Press.

Balaguer, M. (1998). *Platonism and anti-platonism in mathematics*. Oxford: Oxford University Press.

Bechtel, W. (1985). Attributing responsibility to computer systems. *Metaphilosophy, 16*(4), 296–306.

Beer, D. (2016). *Metric power*. London: Palgrave Macmillan.

Bernal, M. (1987). *Black Athena, The Afroasiatic roots of Classical Civilisation* (Vol. 1). London: Free Association Books.

Bishop, A. J. (1988). *Mathematical enculturation*. Dordrecht: Kluwer Academic Publishers.

Cadwalladr, C. (2017). The great British Brexit robbery: How our democracy was hijacked. https://www.theguardian.com/technology/2017/may/07/the-great-british-brexit-robbery-hijacked-democracy. Accessed 12 Oct 2019.

Chiodo, M. & Bursill-Hall, P. (2018). Ethics in mathematics discussion Paper 1/2018. https://ethics.maths.cam.ac.uk/assets/dp/18_1.pdf. Accessed 20 Oct 2019.

Chiodo, M. & Vyas, R. (2018). The role of ethics in a mathematical education: A whitepaper. https://ethics.maths.cam.ac.uk/assets/dp/19_1.pdf. Accessed 20 Oct 2019.

Corfield, D. (2020). Michael Harris on virtues of priority. Posted March 19, 2020. The n-Category Café. https://golem.ph.utexas.edu/category/2020/03/michael_harris_on_virtues_of_p.html. Accessed 5 July 2020.

Detlefsen, M., & Arana, A. (2011). Purity of methods. *Philosophers' Imprint, 11*(2), 1–20.

Devine, P. E. (2013). The concept of tradition: A problem out of MacIntyre. *Reason Papers, 35*(1), 107–123.

Dunne, J. (2003). Arguing for teaching as a practice: A reply to Alasdair MacIntyre. *Journal of Philosophy of Education, 37*(2), 353–369.

EiM (2018). The Cambridge University ethics in mathematics project. https://ethics.maths.cam.ac.uk/. Accessed 24 July 2019.

Ernest, P. (1998). *Social constructivism as a philosophy of mathematics*. Albany, New York: SUNY Press.

Ernest, P. (2009). The philosophy of mathematics, values and Keralese mathematics. In P. Ernest, B. Greer, & B. Sriraman (Eds.), *Critical issues in mathematics education* (pp. 189–204). Charlotte, NC: Information Age Publishing.

Ernest, P. (2016). Mathematics and values. In B. Larvor (Ed.), *Mathematical cultures* (pp. 189–214). Cham: Springer.

Ernest, P. (2018). The ethics of mathematics: Is mathematics harmful? In P. Ernest (Ed.), *The philosophy of mathematics education today* (pp. 187–216). Cham: Springer.

Ernest, P. (2019). Privilege, power and performativity: The ethics of mathematics in society and education. *The Philosophy of Mathematics Education Journal* No. 35. http://socialsciences.exeter.ac.uk/education/research/centres/stem/publications/pmej/pome35/index.html. Accessed 13 Oct 2020.

Ferreirós, J. (2016). Purity as a value in the German-speaking area. In B. Larvor (Ed.), *Mathematical cultures: The London meetings 2012–2014* (pp. 215–234). Birkhäuser: Basel.

Floridi, L., & Sanders, J. W. (2004). On the morality of artificial agents. *Minds and Machines, 14*(3), 349–379.

Franssen, M., Lokhorst, G.-J. & van de Poel, I. (2019). Philosophy of technology. *The Stanford Encyclopedia of Philosophy*. https://plato.stanford.edu/archives/fall2018/entries/technology/. Accessed 5 July 2020.

Gorur, R. (2016). Seeing like PISA: A cautionary tale about the performativity of international assessments. *European Educational Research Journal, 15*(5), 598–616.

Grigutsch, S. & Törner, G. (1998). *World views of mathematics held by university teachers of mathematics science* (Schriftenreihe des Fachbereichs Matematik, Preprint 420). Duisburg: Gerhard Mercator University. Online via http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.46.3770. Accessed 20 Oct 2019.