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Accurate Calculation of the Miller Effect on the Frequency Response and the Input and Output Impedances of Feedback Amplifiers

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***Abstract*—The Miller effect on the frequency response and the input and output impedances of feedback amplifiers may now be calculated using a new approach. Its foundation is the Feedback Decomposition Theorem, an extension of Miller's Theorem that was recently stated. By analysis and SPICE simulations, application results achieved using this method are compared to those of Miller's Theorem application, demonstrating its correctness.**

***Index Terms*—Circuit theory, feedback amplifiers, impedance, poles and zeros, simulation, two port methods.**

I. Introduction

MILLER effect is the most popular application of Miller's Theorem [1], mostly concerning the effect of feedback on input impedance and on frequency response of an amplifier. As known, this was first analyzed by J. M. Miller [2] for three-electrode vacuum tubes, but remains valid for all types of amplifiers. The main reason of Miller effect's popularity is the strong intuitive character of the "decomposition" of the feedback network to the known two "reflected" networks, one to the input and one to the output of the main amplifier. However, the feedback amplifier characteristics determined by the usual Miller effect approach, based on Miller's Theorem, have been proved inaccurate, leading to completely different approaches [3] for attaining accurate calculations.

It has been proved that, "although pole splitting occurs with the insertion of Miller capacitor, under certain conditions both poles could move to lower frequencies together" [4]. Therefore, it is important to accurately calculate both the dominant pole and the next pole, for confronting issues like transient response analysis and stability analysis of feedback amplifiers, "especially if a strong feedback loop is closed" [3]. However, the key point is to preserve the strong intuitive character of the "decomposition" of the feedback network into two networks, to the input and output of the main amplifier and be able to carry out calculations based on such an equivalent circuit. Besides gain frequency response, it is also important to accurate-

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ly calculate both input and output impedances for impedance matching purposes. Thus, an equivalent circuit yielded from feedback network's "decomposition" should permit such cal- culations. Several contributions [5], [6], [7], [8], warn about Miller's Theorem use for the output impedance calculation, leading either to bypassing the problem through other calcula- tion methods, or to its calculation straight from the original circuit. Output impedance calculation, under the "feedback decomposition" concept, is claimed to be valid when using the Feedback Decomposition Theorem [9] and it remains to be confirmed by application. The Feedback Decomposition Theo- rem (FDT) [9], presented as evolution of Miller's Theorem, yields a true equivalent to the original circuit, while retaining the Miller's Theorem concept of decomposing the feedback network.

The present contribution proposes a new method for the ac- curate calculation of Miller effect on the frequency response (pole and zeros determination) and on the input and output impedances of feedback amplifiers, based on the FDT. First, a brief presentation of the FDT is given for the cases of forward and reverse calculations, both for unilateral and bilateral basic amplifiers (Section II). Then, based on a generic single stage feedback amplifier, Miller effect on the frequency response is for the first time evaluated using both Miller's Theorem and FDT, concluding that the FDT approach is more accurate (Section III). Miller effect on input and output impedances of a feedback amplifier is also considered for both circuits resulting after Miller's Theorem and FDT applications. It is shown, by analysis and SPICE simulation, that the FDT approach pro- vides a more accurate input impedance calculation, while it is the only possible to provide reverse calculations, e.g. output impedance (Section IV). Section V summarizes the conclu- sions.

II. The Feedback Decomposition Theorem

The Feedback Decomposition Theorem (FDT) aims to sim- plify feedback analysis by simplifying the four basic topolo- gies of single loop feedback amplifiers [9]. It is based on two- port theory and uses the notation of generalized χ-parameters found in [10].

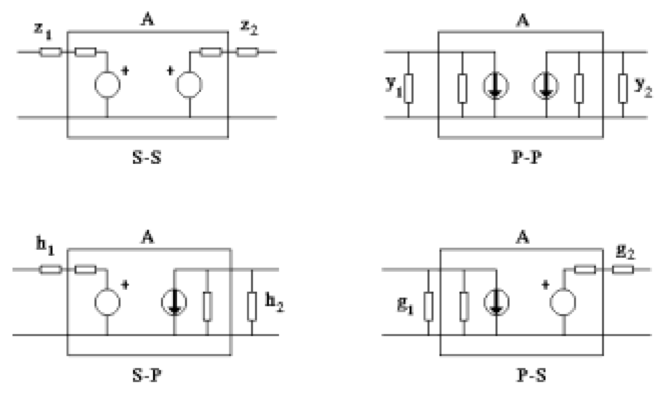
FDT claims that if we have two linear two-port networks, A and B, interconnected in such a way to present the same gener- alized input independent variables and the same generalized output independent variables, then the interconnection of A and B two-ports is equivalent to the connection of two Feed-back Decomposition One-ports (FDOs), O_1 and O_2, to the input and output ports of the A two-port, respectively. This equivalent representation is defined as the Feedback Decom-position Equivalent (FDE). The specific nature of the two FDOs O_1 and O_2 depends on the bilateral/unilateral character of the A two-port and on the considered signal flow direction, as shown on Fig.1. The one-ports that contain no source are given by:

$$\chi_{i,f} = (\chi_{ii})_B + (\chi_{ij})_B A_f^{(-1)^j} \quad (1)$$

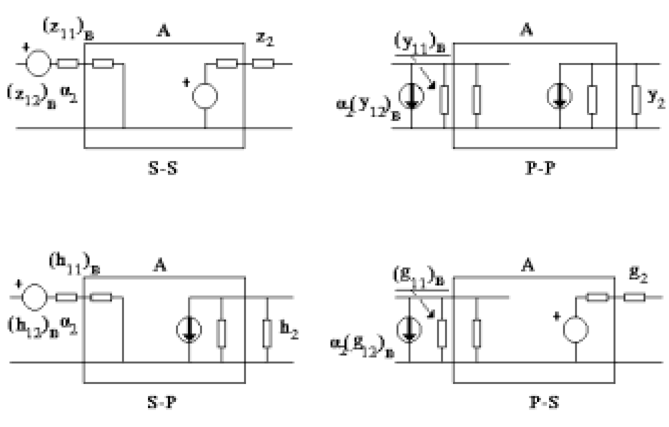
$$\chi_{i,r} = (\chi_{ii})_B + (\chi_{ij})_B A_r^{(-1)^i} \quad (2),$$

where i, j=1,2 with i≠j, and $A_f = -\dfrac{\chi_{21}}{\chi_{22} + \chi_L}$, $A_r = -\dfrac{\chi_{12}}{\chi_{11} + \chi_S}$ with $\chi_{ij} = (\chi_{ij})_A + (\chi_{ij})_B$ and $\chi \in \{z, y, h, g\}$.

Note that, A_f and A_r are given by FDT and they are part of FDOs expressions, so one does not have to analyze the circuit, prior to FDT's application, in order to determine them.



(a)



(b)

Fig. 1. (a) FDE's special case for forward and reverse signal flow, if the A two-port is bilateral, and for forward signal flow only, if the A two-port is unilateral. (b) FDE's special case for reverse signal flow, if the A two-port is unilateral, also valid for forward calculations.

III. Miller Effect on Gain Transfer Function

A simple, generic feedback amplifier circuit that is used to study the effect of the feedback capacitance (known also as Miller capacitance), considered between input and output of a feedback amplifier, is shown in Fig. 2.

According to the traditional approach of Miller's Theorem application, Miller capacitance is decomposed into two capaci-tances C_{M1} and C_{M2} to the input and output, respectively, as shown in Fig. 3, with:

$$C_{M1} = C_f \cdot (1 - A_v) \quad (3),$$

$$C_{M2} = C_f \cdot (1 - 1/A_v) \quad (4),$$

where $A_v = V_2 / V_1$.

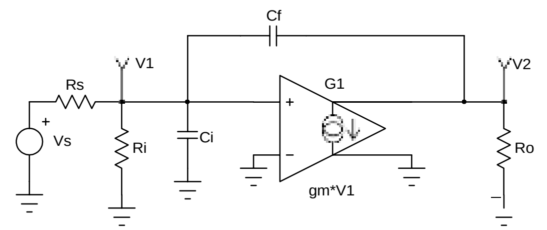


Fig. 2. A generic single stage feedback amplifier.

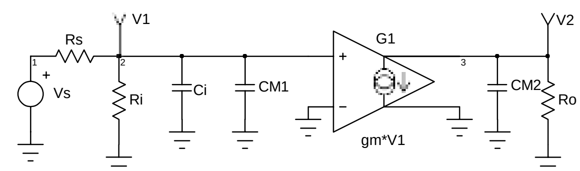


Fig. 3. The circuit of Fig. 2. after the application of Miller's Theorem.

Normally, A_V should be the closed loop forward gain of the feedback amplifier, but this is not clearly stated in Miller's Theorem [5], [1]. However, in most cases of Miller's Theorem application, A_V is assumed to be the dc open loop gain. The reason is that if the closed loop gain is calculated, it clearly depends on the feedback network -the feedback capacitor in this case- and this violates the validity condition of Miller's Theorem, requiring an independent of the feedback network gain [1], [11], [12], [4].

Therefore, $A_V = -g_m R_o$ and substituting to (3) we have:

$$C_{M1} = C_f (1 + g_m R_o) \quad (5),$$

while from (4) we have:

$$C_{M2} = C_f (1 + 1/g_m R_o) \quad (6).$$

Performing simple voltage divider logic calculations on the circuit of Fig. 3, we obtain voltage gain as:

$$G_V = \frac{V_2}{V_s} = \frac{V_1}{V_s} \cdot \frac{V_2}{V_1} \quad (7),$$

with $$\frac{V_1}{V_s} = \frac{1}{1 + \frac{R_s}{R_i} + sR_s(C_i + C_{M1})} \quad (8)$$

and $$\frac{V_2}{V_1} = \frac{-g_m R_o}{1 + sR_o C_{M2}} \quad (9).$$

Thus, substituting (8) and (9) to (7) we have:

$$G_V = \frac{1}{1+R_s/R_i + sR_s(C_i + C_{M1})} \frac{-g_m R_o}{1+sR_oC_{M2}} \tag{10}$$

or using (5) and (6):

$$G_V = \frac{1}{1+R_s/R_i + sR_s\left[C_i + C_f\left(1+g_mR_o\right)\right]} \cdot \frac{-g_mR_o}{1+sR_oC_f\left(1+1/g_m\cdot R_o\right)} \tag{11}$$

From (11), (10) and (9) we conclude that, using Miller's Theorem, Miller effect on the calculated voltage gain is the shifting of the "input" pole and the appearance of an output pole. Note that (11) has no zeros.

Now, applying FDT to analyze Miller effect on the (forward) voltage gain of the circuit of Fig. 2, we first interpret the circuit as a two-port connection, as shown in Fig. 4.

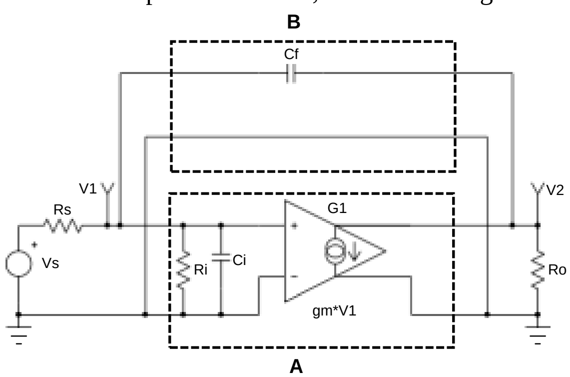


Fig. 4. The circuit of Fig. 2 as a parallel-parallel connection of A and B two-ports.

Then, applying the FDT for forward calculations, we calculate the Feedback Decomposition One-ports that have to be connected to the input and output of the basic amplifier, as [9]: $y_{1,f} = (y_{11})_B + (y_{12})_B A_f$ and $y_{2,f} = (y_{22})_B + (y_{21})_B A_f^{-1}$, with $A_f = \frac{V_2}{V_1} = -\frac{y_{21}}{y_{22}+y_L}$ and $y_{ij} = (y_{ij})_A + (y_{ij})_B$ (we use y-parameters due to the parallel-parallel connection of A and B two-ports). Applying two-port definitions and properties [10], we easily obtain Y-matrices for the feedback network and the basic amplifier, respectively:

$$\mathbf{Y}_B = sC_f\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix},\ \mathbf{Y}_A = \begin{bmatrix}(sC_iR_i+1)/R_i & 0\\ g_m & 0\end{bmatrix}.$$

Then, in a straightforward way, we have:

$A_f = -\frac{g_m - sC_f}{sC_f + 1/R_o}$ and finally the input one-port is:

$$y_{1,f} = \frac{1}{\left(g_m + R_o^{-1}\right)^{-1} + R_o^{-1}/sC_f\left(g_m+R_o^{-1}\right)} \tag{12}$$

which is the connection of a resistor $R_{1,f}$ and a capacitor $C_{1,f}$ in series, with $R_{1,f} = \frac{1}{g_m + R_o^{-1}}$ and $sC_{1,f} = \frac{sC_f\left(g_m+R_o^{-1}\right)}{R_o^{-1}}$, while the output one-port is:

$$y_{2,f} = \frac{1}{-\frac{1}{g_m+R_o^{-1}} + \frac{g_m}{sC_f\left(g_m+R_o^{-1}\right)}} \tag{13}$$

which is the connection of a resistor $R_{2,f}$ and a capacitor $C_{2,f}$ in series, with $R_{2,f} = -\frac{1}{g_m+R_o^{-1}}$ and $sC_{2,f} = \frac{sC_f\left(g_m+R_o^{-1}\right)}{g_m}$.

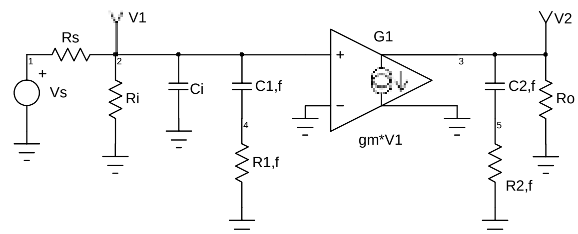


Fig. 5. The circuit of Fig. 2 after the application of FDT, for forward calculations.

The equivalent circuit resulting from the application of the FDT, i.e. the FDE, is illustrated in Fig. 5. Following the same analysis method, we calculate the voltage gain by calculating the factors shown in (7), to give a clear idea of the input and output contribution to the transfer function. By simple voltage divider logic calculations on the circuit of Fig. 5, we obtain (14) (bottom of page) and (15):

$$\frac{V_2}{V_1} = -g_mR_o\frac{1+sR_{2,f}C_{2,f}}{1+sC_{2,f}\left(R_{2,f}+R_o\right)} \tag{15}$$

Substituting (12) and (13)'s components to (14) and (15), we express (14) and (15) in terms of the components of the original circuit (Fig. 2), as in (16) (bottom of page) and (17):

$$\frac{V_2}{V_1} = -g_mR_o\frac{1-s\left(C_f/g_m\right)}{1+sC_fR_o} \tag{17}$$

Finally, from (7), (16) and (17), and after the $1+sC_fR_o$ factor cancellation from nominator and denominator (pole-zero cancellation), we have (18) (top of next page):

$$\frac{V_1}{V_s} = \frac{1+sR_{1,f}C_{1,f}}{1+\frac{R_s}{R_i}+s\left[C_{1,f}\left(R_{1,f}+\frac{R_s}{R_i}R_{1,f}+R_s\right)+C_iR_s\right]+s^2C_{1,f}C_iR_{1,f}R_s} \tag{14}$$

$$\frac{V_1}{V_s} = \frac{1+sC_fR_o}{1+\frac{R_s}{R_i}+s\left[C_fR_o+C_fR_s\left(1+R_o\left(g_m+R_o^{-1}\right)\right)+C_iR_s\right]+s^2C_fC_iR_oR_s} \tag{16}$$

$$G_V = \frac{V_2}{V_s} = -g_m R_o \frac{1-s\left(C_f / g_m\right)}{1+\frac{R_s}{R_i}+s\left[C_f R_o + C_f R_s\left(1+R_o\left(g_m + R_s^{-1}\right)\right)+C_i R_s\right]+s^2 C_f C_i R_o R_s} \quad (18),$$

which is exactly the voltage gain if calculated directly from the original circuit of Fig. 2 [3].

Apart from the fact that Miller's Theorem approach ignores the zero of the voltage gain, even the poles of the original circuit are not accurately calculated as SPICE calculations prove. Using the same values as in [3] (R_s= 2kohms, R_i= 2kohms, C_i= 100pF, g_m=0.01, R_o=10kohms, and C_f=1pF), the simulation of the original circuit gives one zero at 10^{10} rad/s and two poles at $-4.85086 \cdot 10^6$ rad/s (dominant pole) and $-2.06149 \cdot 10^8$ rad/s. We also performed a SPICE pole-zero analysis on the original circuit to find the pole-zero contribution of the two factors present in (7), i.e. "input" and "output" contribution to circuit's poles and zeros. We found that the input contributes one zero at $-1 \cdot 10^8$ rad/s and two poles at $-4.85086 \cdot 10^6$ rad/s and $-2.06149 \cdot 10^8$ rad/s, while the output contributes with one zero at $-1 \cdot 10^{10}$ rad/s and one pole at $-1 \cdot 10^8$ rad/s, with an apparent pole-zero cancellation at $-1 \cdot 10^8$ rad/s. We verified these results, by analyzing the original circuit, in terms of the input and output factors contribution, as implied by (7).

On the other hand, SPICE simulation on the circuit of Miller's Theorem (Fig. 3) yields only two poles at $-4.97512 \cdot 10^6$ rad/s (dominant pole) and $-9.90099 \cdot 10^7$ rad/s. Clearly, dominant pole is shifted by 2.6%, while the second pole is wrongly calculated as it presents 52% deviation from the original circuit's second pole, closer to the dominant pole. Their percentage difference depends on the actual values of the original circuit [8]. But, while the dominant pole results from the Miller effect on the input of the basic amplifier, as referred in (8), simulation on Fig. 3 circuit proved that the second pole does not result from the Miller effect on the output of the basic amplifier as (9) implies. Moreover, there is a small difference between the simulated ($9.90099 \cdot 10^7$ rad/s) and the calculated from (9) ($1.01 \cdot 10^8$ rad/s) second pole's position. In fact, simulation gives one zero at at $-9.90099 \cdot 10^7$ rad/s and two poles, at $-4.97512 \cdot 10^6$ rad/s and $-9.90099 \cdot 10^7$ rad/s, contributed by the input, while the output contributes one zero at $-9.90099 \cdot 10^7$ rad/s. As expected, the overall gain presented a pole-zero cancellation at $-9.90099 \cdot 10^7$ rad/s.

Finally, the simulation of the FDE circuit (Fig. 5) gives exactly the same results with the original circuit, both for the overall and the input and output pole-zero analysis. Of course, the simulation verified the pole-zero pair at $-1 \cdot 10^8$ rad/s, which is predicted by (16), (17), and (18), in the same way as in the simulation and analysis of the original circuit. Thus, using the FDT, there is no need to be cautious about any error analysis as when using Miller's Theorems [8].

Based on the above, the FDT approach yields an equivalent to the original circuit that leads to accurate determination of Miller effect on the input, the output and the overall voltage gain, while Miller's Theorem approach does not yield an equivalent circuit. Only Miller effect on the dominant pole of the basic amplifier is interpreted by Miller's Theorem approach, although with some inaccuracy, while Miller effect on the overall frequency response and on the input and output of the basic amplifier is not successfully interpreted.

IV. Miller Effect on Input and Output Impedance

Following Miller's Theorem approach we computed the input impedance (Fig. 3) to be:

$$Z_{in} = R_s + \frac{1}{R_i^{-1} + s\left[C_i + C_f\left(1+g_m R_o\right)\right]} \quad (19)$$

and the output impedance (short circuiting V_s and replacing R_o with an ideal test voltage source) to be:

$$Z_{out} = \frac{1}{sC_f\left(1+1/g_m R_o\right)} \quad (20).$$

Based on the equivalent circuit of Fig. 5, we calculated the input impedance as:

$$Z_{in} = R_s + \frac{1}{R_i^{-1} + sC_i + \frac{sC_f(g_m + R_o^{-1})}{sC_f + R_o^{-1}}} \quad (21),$$

which we verified it as identical to the Z_{in} calculated directly from the original circuit of Fig. 2.

For output impedance calculation, the circuit of Fig. 5 is not suitable. According to FDT, a reverse calculations equivalent circuit is needed, which in our case of a unilateral amplifier is the one depicted in Fig. 6. This is because if we had used a circuit like the one of Fig. 5, we would have made the input completely independent from the output, destroying the equivalence to the original circuit in reverse calculations [9].

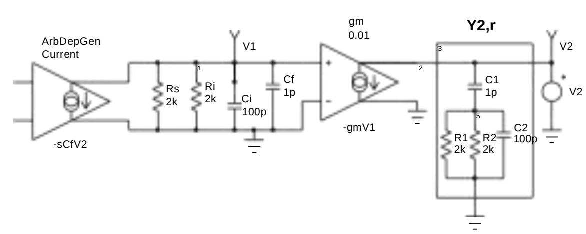


Fig. 6. The circuit of Fig. 2 after the application of FDT, for reverse calculations. This circuit is referred to as the Feedback Decomposition Equivalent (FDE) for reverse calculations.

So, applying FDT for reverse calculations to the circuit of Fig. 2, we add the output one-port to the output and leave the input circuit of the B two-port to the input. As described in [9], the output one-port is:

$$y_{2,r} = \left(y_{22}\right)_{\mathrm{B}} + \left(y_{21}\right)_{\mathrm{B}} A_r, \text{ with } A_r = \left.\frac{V_1}{V_2}\right|_{\substack{\varepsilon_s = 0 \\ \chi_s \neq 0}} = -\frac{y_{12}}{y_{11} + y_S} \text{ and}$$

$y_{ij} = (y_{ij})_{\mathrm{A}} + (y_{ij})_{\mathrm{B}}$.

Then, in a straightforward way, we have:

$A_r = \dfrac{sR_i R_s C_f}{R_i + R_s + sR_i R_s (C_i + C_f)}$, while the output one-port is:

$$y_{2,r} = \frac{sC_f (R_i + R_s + sC_i R_i R_s)}{R_i + R_s + sR_i R_s (C_i + C_f)} \text{ or}$$

$$y_{2,r} = \frac{1}{\dfrac{1}{R_i^{-1} + R_s^{-1} + sC_i} + \dfrac{1}{sC_f}} \quad (22),$$

which is the connection of a capacitor $C_1 = C_{1,f}$ in series to the parallel connection of two resistors $R_1 = R_i$, $R_2 = R_s$ and a capacitor $C_2 = C_i$, as depicted on Fig. 6.

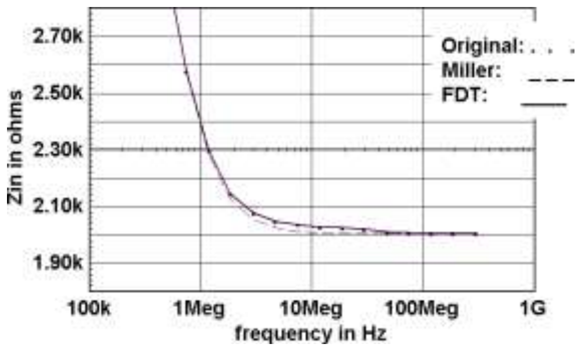


Fig. 7. The input impedance of the circuit of Fig. 2 simulated, using the original circuit and the circuits derived from the application of Miller's Theorem and FDT. Maximum difference of about 1.2% between the original and Miller circuit's input impedances.

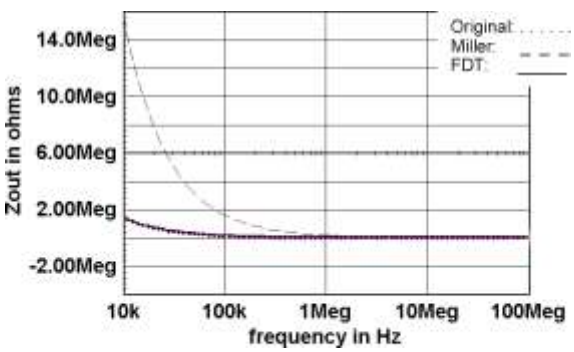


Fig. 8. The output impedance of the circuit of Fig. 2 simulated, using the original circuit and the ones derived from the application of Miller's Theorem and FDT.

Based on the equivalent circuit of Fig. 6, we calculated the output impedance as follows: $Z_{out} = Y_{out}^{-1} = (y_{2,r} + y_{gm})^{-1}$, with $y_{gm} = g_m V_1 / V_2$.

But $V_1 = sC_f \left[R_i^{-1} + R_s^{-1} + s(C_i + C_f)\right]^{-1} V_2$ and Z_{out} finally yields:

$$Z_{out} = \frac{R_i + R_s + sR_i R_s (C_i + C_f)}{sC_f (R_i + R_s + g_m R_i R_s) + s^2 R_i R_s C_i C_f} \quad (23),$$

which we cross-checked it as identical to the Z_{out} calculated directly from the original circuit of Fig. 2.

Comparing the two approaches, it is apparent that (19) and (21) give the same dc input impedances, equal to $R_s + R_i$, but somewhat different frequency response. Their percentage difference depends on the actual values of the original circuit [8]. From (21) and (23) we can conclude that the output impedance derived by the Miller's Theorem approach has no relevance to the original circuit's output impedance and consequently Miller effect on the output impedance is neither accurately nor qualitatively represented. This is mainly due to the fact that the Miller's Theorem application does not give an equivalent to the original circuit for reverse calculation. In fact, Miller's Theorem does not discriminate between forward and reverse calculation at all. Apart from verifying Z_{in} and Z_{out} to the ones resulting directly from the original circuit of Fig. 2, we performed SPICE simulation as well (using again the same values -shown on Fig. 6- as in simulations of section III). Obtained results confirming the above are shown in Fig. 7 and Fig. 8.

V. Conclusion

The following conclusions emerge:

1. FDT approach gives an equivalent to the original circuit that leads to accurate determination of Miller effect on the input, output and overall voltage gain.

2. Only Miller effect on the dominant pole of the basic amplifier is interpreted by Miller's Theorem approach, although with some inaccuracy.

3. Miller effect on input and output impedances is accurately derived by the FDT approach, providing an equivalent circuit for both forward and reverse calculations. On the other hand, Miller's Theorem approach provides only an approximate calculation of the input impedance, while fails to estimate output impedance.

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